

2012

Essays on financial institutions and instability

Yu Jin

Iowa State University

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Essays on financial institutions and instability

by

Yu Jin

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Economics

Program of Study Committee:

David M. Frankel, Major Professor

Joydeep Bhattacharya

Harvey E. Lapan

Gary M. Lieberman

Rajesh Singh

Iowa State University

Ames, Iowa

2012

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DEDICATION

I would like to dedicate this thesis to my wife Jing without whose support I would not have been able to complete this work. I would also like to thank my friends and family for their loving guidance and financial assistance during the writing of this work.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	v
ABSTRACT	vi
CHAPTER 1. BANK MONITORING AND LIQUIDITY IN THE SEC-	
ONDARY MARKET FOR LOANS	1
1.1 Introduction	1
1.2 A Dynamic Lending Model	4
1.2.1 Relationship Lending	7
1.2.2 The Secondary Market for Loans	10
1.2.3 Transactional Lending	11
1.3 Decentralized Credit Market	12
1.3.1 Interbank Relationships	15
1.3.2 Multiple Equilibria	18
1.4 Private Signals: Unique Equilibrium	19
1.4.1 Solving the Model	19
1.4.2 The Loan Market Analysis	21
1.5 Conclusion	24
CHAPTER 2. SECURITIZATION AND LENDING COMPETITION	26
2.1 Introduction	26
2.2 The Model	30
2.2.1 Timing	34
2.2.2 Payoffs	39
2.2.3 Summary	40

2.3	Base Model: No Securitization	40
2.4	Full Model	41
2.5	Illustrations	50
2.5.1	The Effects of Securitization	50
2.5.2	Higher Securitization Profits for the Local Bank	53
2.5.3	Higher Securitization Profits for the Remote Bank	55
2.5.4	Efficiency Effects	55
2.6	Related Literature	56
2.6.1	Lending with Securitization	56
2.6.2	Security Design	58
2.6.3	Lending Competition with Adverse Selection	61
2.7	Conclusions	62
CHAPTER 3. CREDIT TERMINATION AND TECHNOLOGY BUBBLES		66
3.1	Introduction	66
3.2	The Model	68
3.3	Supply and Demand of Loanable Funds	70
3.3.1	Short-Term Loan Contract	70
3.3.2	Long-Term Loan Contract	71
3.3.3	Credit Market Equilibrium	75
3.4	Shocks and Technology Bubbles	77
3.4.1	Technology Shocks	77
3.4.2	Technology Bubbles	78
3.5	Credit Termination and the Dot-Com Bubble - An Example	79
3.6	Conclusion	81
APPENDIX A. ADDITIONAL MATERIAL FOR CHAPTER 1		83
APPENDIX B. ADDITIONAL MATERIAL FOR CHAPTER 2		92
APPENDIX C. ADDITIONAL MATERIAL FOR CHAPTER 3		102
BIBLIOGRAPHY		106

ACKNOWLEDGEMENTS

I would like to take this opportunity to express my thanks to those who helped me with various aspects of conducting research and the writing of this dissertation. First and foremost, Dr. David M. Frankel for his guidance, patience and support throughout this research and the writing of this dissertation. His insights and words of encouragement have often inspired me and renewed my hopes for completing my graduate education. I would like to thank my committee members for their efforts and contributions to this work: Dr. Joydeep Bhattacharya, Dr. Harvey E. Lapan, Dr. Gary M. Lieberman and Dr. Rajesh Singh. I would also like to thank Dr. Cheng Wang and Dr. Charles Z. Zheng for their guidance throughout the initial stages of my graduate career.

ABSTRACT

Influenced by the recent, ongoing financial crisis spreading across the world's economies, my dissertation studies aspects of the connections between securitization - originating and selling loans - in the banking sector and economic instability.

In the first chapter, "Bank Monitoring and Liquidity in the Secondary Market for Loans", I study transactional loans and traditional-relationship loans in a dynamic lending model. In the model, since transactional loans are easier to resell, a bank's benefit from transactional lending over relationship lending is increasing in secondary market loan liquidity (investors' willingness to pay). The relative payoff is also increasing in the proportion of banks that choose transactional lending because lower quality borrowers prefer transactional lenders, who monitor them less. When liquidity rises above a given threshold, all banks switch to transactional lending. However, greater liquidity also increases the economy-wide default risk since banks reduce their monitoring effort. If the latter effect is strong enough, securitization can lower welfare.

The previous study suggests that the problems in securitization may come from information asymmetry in both the primary and secondary loan markets. My second chapter, "Securitization and Lending Competition" (with David Frankel), studies the effects of securitization on interbank lending competition when banks see private signals of local applicants' repayment chances. We find that if banks cannot securitize, the outcome is efficient: they lend to their most creditworthy local applicants. With securitization, banks lend also to remote applicants with strong observables in order to lessen the lemons problem they face in selling their securities. This reliance on observables is inefficient and raises the conditional and unconditional default risk.

Finally, Chapter 3, "Credit Termination and Technology Bubbles", studies the financial instability from a different angle. I consider a credit cycles model in which firms face technology

shocks to the riskiness of different types of projects. The new project arriving is more attractive to the firms but even riskier. The riskiness of the new project is not observed by banks as occurred during the technology bubbles. After observing a higher default rate, banks deny future loans to entrepreneurs more often in order to affect their choice of projects ex ante. The model is used to explain the boom-and-bust of the dot-com bubble in the late 1990s.

CHAPTER 1. BANK MONITORING AND LIQUIDITY IN THE SECONDARY MARKET FOR LOANS

1.1 Introduction

Securitization of bank loans - originating and selling loans - is driven by the innovation of structured finance products and the development of a secondary market for loans. What are the consequences of the development of this secondary loan market? Since loans can be easily sold to third parties, this innovative process of transactional lending may dilute a bank's incentive to monitor borrowers. In the recent financial crisis in the U.S., concerns about the credit quality of transactional loans led to a "run" on banks: a drop in total demand of loan-backed securities by creditors (Ivashina and Scharfstein [36]).

In this chapter, I propose a dynamic lending model to study the impact of securitization on a bank's incentive to monitor borrowers. In this model, a bank's relative payoff from transactional lending versus relationship lending depends on investors' willingness to pay for securitized loans, which depends on a random fundamental - liquidity.¹ In addition, the relative benefit of choosing a transactional technology is increasing in the proportion of banks that do so, because each bank will face fewer low quality borrowers, which reduces the urgency to build close relationships to monitor borrowers.

Consistent with the stylized fact observed, the model shows that the secondary loan market will experience a boom after an improvement of liquidity. One heavily studied market is the U.S. secondary market for syndicated loans. The U.S. secondary loan market grew a lot before the fall of 2008 and the average annual growth rate was about 26% during 1991-2007 according

¹Banks liquidate their loans and investors bid for the loan portfolios in the secondary market for loans. The definition of liquidity follows Shleifer and Vishny [60] in which asset illiquidity is the difference between asset price and value in the best use.

to Thomson Reuters LPC Traders Survey. But after that, the secondary loan market crashed followed by a fire sale - the sale at discounted prices - of transactional loans. There is no convincing explanation of why.

In addition, the model shows that transactional loans, which can be traded in a secondary market, may replace traditional bank loans when the costs of building relationships between banks and borrowers are sufficiently high. The model thus points out a potential problem: banks have less incentive to monitor borrowers when securitization passes bad loans to unsuspecting investors. Bad loans have been sold to final investors who have less information on loans' default risk. In turn, while a positive liquidity shock (for example, an investor may have a surprise portfolio need) raises the proportion of loans sold in the secondary market, it may also lower total output since banks monitor less. Hence, restrictions on securitization may raise social welfare if the effect of defaulting is dominant.

In this model, bank loans are multi-period contracts which are designed to make borrowers focus on long-term returns and thus mitigate moral hazard. More precisely, in Section 1.2, I consider a benchmark model in which a borrower's project occasionally fails. There are two types of projects: good and bad. The former has a higher chance of success, but a lower private benefit for the borrower. There are two monitoring technologies. A bank may exert a costly effort to discover the type of project.² I refer to this as the active technology. Alternatively, the bank may decide whether or not to renew its loan based solely on the borrower's output in the first period. I call this the passive technology.

Unlike previous lending models, I introduce a secondary market for transactional loans in which investors buy loan portfolios. Since the bank's effort is unobservable, if the bank resells a loan, it loses its incentive to monitor borrowers. The secondary loan market takes this into account, and does not value the extra effort to build relationships between banks and firms. Hence, relationship loans will not be traded in the secondary market. Transactional loans can be traded easily because the bank only requires public observed output information and punishes the borrower whenever her project fails in this case.³

²Similar credit-worthiness monitoring which provides information of borrowers is discussed in Nalebuff and Scharfstein [51] and Broecker [8].

³Transactional loans share features of arm's length lending (Boot and Thakor [7]). In this chapter, banks

I then present, in Section 1.3, a model in which banks compete to attract borrowers with good projects. Each borrower knows about the banks' technology choices and searches for a bank that uses her preferred technology. Therefore, banks' technology choices affect the mix of good and bad projects that each of them will get from a given technology. Due to this coordination externality among banks, a bank's optimal technology choice may depend on its opponents' behavior. There may be multiple equilibria, which may reduce the prediction power of the model. Hence in Section 1.4, I extend the model to the case in which banks have slightly noisy private signals of the state of liquidity. In this case, global games techniques are used to show that there is a unique equilibrium: a bank will offer transactional lending if its signal exceeds a common threshold, and relationship lending otherwise. In this unique equilibrium, a small positive liquidity shock can lead to a large increase in transactional lending, with a resulting increase in default risk.

Existing theories have studied carefully the cost and benefit of relationship lending compared with transactional lending. To mitigate conflicts of interest between lenders and borrowers, relationship loans provide the incentive effects of reputation (Diamond [21]) and promise to make credit available in the future (Boot, Creenbaum, and Thakor [5]). But relationship lenders have bargaining power over the borrowers' profits (Rajan [56]; Sharpe [58]). Boot and Thakor [7] further discuss whether or not relationship lenders survive competitive pressures from transactional lenders, such as mutual funds and investment banks. However, existing theories ignore the embedded instability problem when lenders can switch from relationship lending to transactional lending. The contribution of this chapter is to emphasize how liquidity shocks in the secondary loan market cause the financial instability.

A basic concern of the securitization process is whether or not it reduces the incentive to conduct credit risk analysis or monitor borrowers (Gorton and Pennacchi [26]). Although it is not conclusive, evidence from the current subprime mortgage crisis suggests that securitization may adversely affect the screening incentive of lenders.

- **Loan portfolio with greater ease of securitization defaults more.** A recent empir-

offer long-term contracts to borrowers in transactional lending as well as relationship lending. The difference lies on a bank's effort to acquire borrower specific information.

ical work by Keys et al. [38] suggests that, conditional on being securitized, the portfolio with greater ease of securitization defaults more than a similar risk profile group with a less ease of securitization.

- **Securitized loans have higher foreclosure rates and lower cure rates.** Piskorski et al. [54] study securitized mortgages issued without a guarantee from GSEs. They compare bank-held loans with securitized loans. Their evidence suggests that the foreclosure rate is lower and the cure rate is higher for bank-held loans. In addition, delinquent securitized loans that are taken back on the bank's balance sheet foreclose at a rate lower than delinquent securitized loans that continue to be securitized.

This chapter connects to these empirical results and shows a mechanism that liquidity shocks in the secondary loan market change the monitoring incentive of banks.

1.2 A Dynamic Lending Model

The lending model lasts two periods, numbered $t = 1, 2$. Both borrowers and lenders are risk neutral. First, I consider a benchmark case in which each borrower can raise funds from a lender assigned randomly in a decentralized primary credit market. In this case, the borrower distribution that each lender faces is independent of its opponents' behavior. One unit of capital costs lenders an amount $D > 1$ in each period.

In addition, I assume a fixed fraction $\ell_0 \in (0, 1)$ of lenders cannot monitor borrowers. The existence of this type of arm's length lending is potentially due to the cost to offer relationship lending. The rest of them, $\ell_1 = 1 - \ell_0$, are lenders (banks hereafter) that can choose which type of lending to offer. Focus on the behavior of banks indexed by $i \in [0, \ell_1]$.

In the primary credit market, banks originate loans in the first period. Initially, a bank can take an action $a \in \{0, 1\}$ (to choose the loan contract type): 0 is defined as relationship loans and 1 as transactional loans, respectively. Relationship loans are similar to traditional commercial bank loans. If a loan defaults, the bank exerts a costly effort to screen the loan and renews it if and only if it is good. In the case of a transactional loan (or an arm's length lending), the bank denies the second-period loan whenever the borrower defaults on her first-

period payment. One important difference from the traditional credit market is the existence of a secondary market in which the lender has an option of selling loan portfolios. The timing of the model is showed in Figure 1.1.

Each borrower needs one unit of capital in each period for an investment opportunity - a project. There are two types of projects: good projects H and bad projects L . In each period, a type $\tau \in \{H, L\}$ project yields either a positive output $\theta > 0$ with probability p_τ or zero output with probability $1 - p_\tau$, where the probability p_H of the good project is strictly larger than the probability p_L of the bad one.

There are two types of borrowers: h and l . Each type $g \in \{h, l\}$ contains a mass $\mu_g > 0$ of borrowers. Let the vector $\mu = (\mu_h, \mu_l)$ represent the measure of each type of borrowers. A type- h borrower can invest in either project, while a type- l borrower only has access to a bad project. Each borrower can choose one project only in period 1. Once the project is chosen, it cannot be changed in the second period.

Each type- h borrower has a reservation utility $v_h = v_0 > 0$ each type- l borrower has a zero reservation utility $v_l = 0$. Each borrower receives a private benefit $b > 0$ from a type- L project. Hence the type- l borrower's participant constraint is trivially satisfied. I further assume that the reservation utility v_0 is less than $p_H^2 \theta$ and the private benefit b is less than $[(p_H^2 - p_L^2) / p_H] v_0$. These two restrictions guarantee that the type- H project is preferred and is feasible to banks.

The long-term loan contracts offered by banks specify a gross interest rate R_t in each period $t = 1, 2$, and the termination condition.⁴ The incentive effects of termination are first discussed in Stiglitz and Weiss [63]. Due to limited liability, borrowers cannot pay more than the project output $0 \leq R_t \leq \theta$. In each period, a borrower must pay either the gross interest rate or her total output, whichever is lower. However, her private benefit is not pledgable to the payment. Therefore, in each period t , the single-period bank payoff from a type $\tau \in \{H, L\}$ project is $v_\tau(R_t) = p_\tau R_t - D$. Finally, I shall assume the type- H project is socially desirable but the

⁴The banks can terminate a defaulting borrower's loan in dynamic contract. Denying a borrower's loan has two positive effects: the bank may terminate a type- l borrower with a bad project; at the same time, it can save incentive rents by punishing a type- h borrower who defaults on the loan. These two effects, which depend on the borrower's distribution, will balance the negative effect of ruling out the type- h borrowers with good projects.

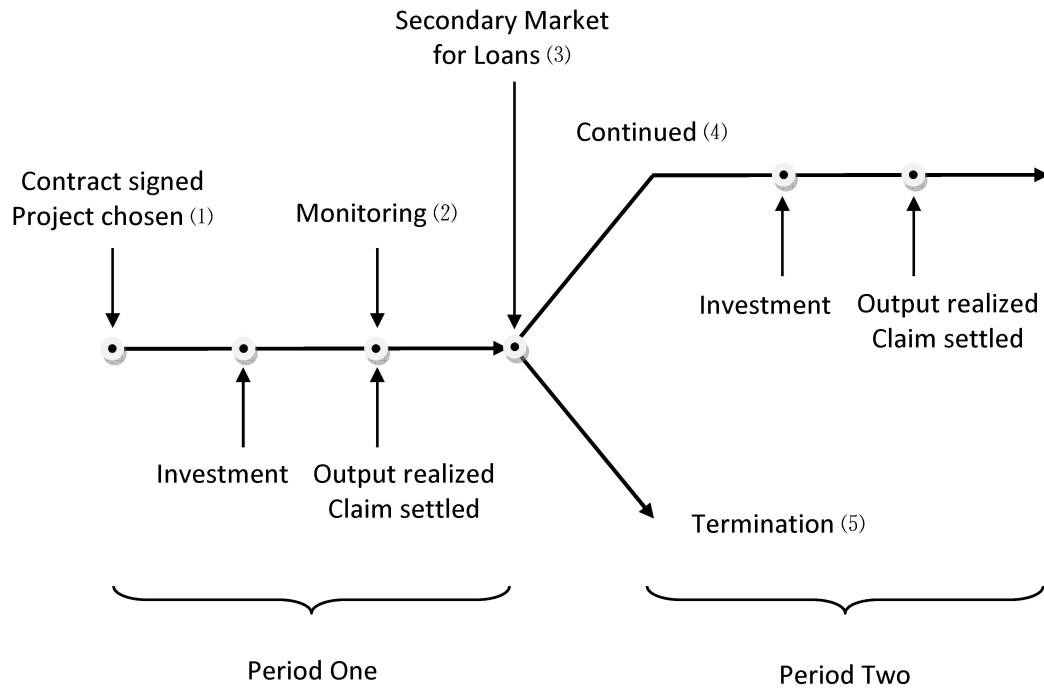


Figure 1.1 The Time-Line of the Model. The model has two periods. (1) At the beginning of the first period, the bank offers long-term loan contracts. Each borrower takes the offer and chooses a project which requires investment in each period. The two parties share the realized output. At the end of the first period, there are two extensions to the basic model: (2) the bank can exert an effort to discover a borrower's chosen project (relationship lending); or (3) the bank can resell the loan portfolio in a secondary market for loans (transactional lending). Observing the new information, the bank can choose whether or not to terminate the second-period loan. (4) If the bank continues the loan, there is a second-period investment and the two parties share the realized second-period output. (5) Otherwise, there is no investment in the second period.

type- L project is not: $p_L\theta < D < p_H\theta$.

1.2.1 Relationship Lending

A bank can use relationship lending to alleviate the moral hazard problem. I will also refer to relationship lending as the active technology. A bank that chooses this technology has the option of paying an amount $m > 0$ to monitor one borrower at the end of period 1. If it monitors, the bank learns the true type of the borrower.⁵ Let $\tilde{\mu}_h = \mu_h p_H$ and $\tilde{\mu}_l = \mu_l p_L$ denote the measure of type- h and type- l borrowers whose projects succeed. Assume the cost m is larger than the constant $\hat{m} \equiv \delta (\tilde{\mu}_h + \tilde{\mu}_l)^{-1} \tilde{\mu}_l D$. Under this condition, the bank will use the active technology to monitor the borrower only when she defaults in period 1.

Let V_t^0 be the bank's payoff in period $t = 1, 2$ if relationship lending is used. The bank's payoff in period 1 is the sum of its project-specific payoffs weighted by the vector μ :

$$V_1^0(R_1; \mu) = \mu_h v_H(R_1) + \mu_l v_L(R_1). \quad (1.1)$$

If the project succeeds in period 1, the investment continues without further investigation. If the project fails in period 1, the bank goes to a monitoring stage. In this stage, the project continues if it is proved to be a type- H project; and the project is terminated if it is proved to be a type- L project. Therefore, the bank's payoff in period 2 is adjusted by the type- l borrower's probability of succeeding p_L :

$$V_2^0(R_2; \mu) = \mu_h v_H(R_2) + \tilde{\mu}_l v_L(R_2). \quad (1.2)$$

Finally, with the active technology, the total cost C is the sum of individual cost m to monitor one borrower,

$$C(m, \mu) = [\mu_h (1 - p_H) + \mu_l (1 - p_L)] m. \quad (1.3)$$

Banks discount future cash flows at a fraction $\delta \in (0, 1)$. Expecting the vector μ of the measure of borrowers, the bank maximizes its expected profit,

$$(RL) \max_{\{R_1, R_2\}} V_1^0(R_1; \mu) + \delta V_2^0(R_2; \mu) - C(m, \mu),$$

⁵Relationship loans are similar to transactional loans except that whether or not to offer the second-period loan is based on the result of evaluation from the active monitoring technology in addition to the first-period outcome. For example, the bank may employ an enlisted loan officer to evaluate a borrower and its decision to deny the investment depends on the loan officer's investigation.

subjects to the IR and IC constraints,

$$(IR) \quad p_H (\theta - R_1) + p_H (\theta - R_2) \geq v_0, \quad (1.4)$$

$$(IC) \quad p_H (\theta - R_1) + p_H (\theta - R_2) \geq p_L (\theta - R_1) + p_L^2 (\theta - R_2) + b. \quad (1.5)$$

The IR (1.4) and IC (1.5) conditions represent the borrower's return requirement and investment choice, respectively. The type- h borrowers move to period 2 for sure instead of with probability p_H in the transactional lending case. The solution of the problem (RL) is summarized in Lemma 1.1. All proofs in this section are in Appendix A.

Lemma 1.1 *In the optimal relationship loan contract, given the monitoring cost $m > \hat{m}$ the bank monitors the borrowers only if their projects fail in the first period. Moreover, the optimal relationship loan is as follows:*

1. *if the project fails, the gross interest rate in each period $t = 1, 2$ is $R_t^{0*} (0) = 0$;*
2. *if the project succeeds, the gross interest rates are $R_1^{0*} (\theta) = \theta$ and $R_2^{0*} (\theta) = \theta - v_0/p_H$.*

Similarly, define $R_1^{0*} \equiv R_1^{0*} (\theta)$ and $R_2^{0*} \equiv R_2^{0*} (\theta)$. The bank maximizes its expected profit by offering the optimal financial contract $\{R_1^{0*}, R_2^{0*}\}$.

Why does the bank monitor the borrowers only if their projects fail in period 1? First, since each borrower's choice of project is private information and cannot be observed *ex ante*, the bank must monitor the project only at the end of period 1 when the output is realized. In addition, the bank will compare the cost of monitoring all projects with the cost of monitoring the projects if they fail in period 1. The bank will do the latter when the individual monitoring cost m is larger than the threshold \hat{m} . It is possible that, when the cost is lower enough, the optimal contract is to monitor projects no matter whether they succeed or fail in period 1. I do not consider this case because we hardly observe such contract in practice. In addition, the main result does not depend on whether or not the bank monitors after a high output.

The result is showed in Figure 1.2. Intuitively, the optimal contract must be in the set satisfying the limited liability, IR and IC constraints. Also, since a bank maximizes its expected profit, the IR constraint is binding. Thus the optimal contract lies on the line representing the

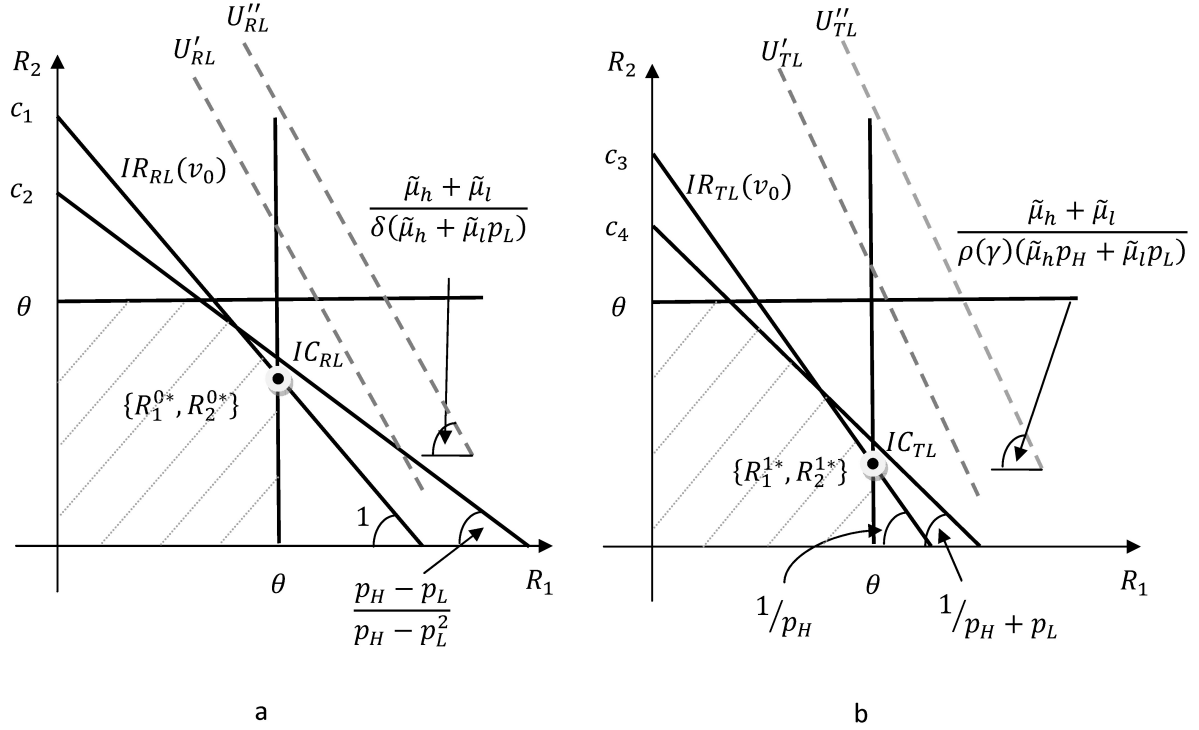


Figure 1.2 The Optimal Relationship (RL) and Transactional Lending (TL), $\{R_1^{a*}, R_2^{a*}\}$. The R_1 -axis and the R_2 -axis are gross interest rates in the first period and second period, respectively. The bank's isoprofit curves are $U'_{RL} < U''_{RL}$, and $U'_{TL} < U''_{TL}$ etc. A feasible interest rate in each period is less than or equal to θ due to limited liabilities. The pair of interest rates is below the incentive compatibility constraint IC_{RL} or IC_{TL} . And in equilibrium, the pair of interest rates lies on the individual rationality constraint $IR_{RL}(v_0)$ or $IR_{TL}(v_0)$ given a promised utility v_0 . Finally, the points $\{R_1^{1*}, R_2^{1*}\}$ and $\{R_1^{0*}, R_2^{0*}\}$, which represent the optimal relationship and transactional lending respectively, lie on the limited liability constraint $R_1 = \theta$ because borrowers are required to pay the interest rate as soon as possible. See Appendix A for details of the parameters.

binding IR constraint. Why is the optimal contract at the right end of this IR constraint? Or why the bank's isoprofit curve is steeper than the IR constraint? Recall that a type- l borrower whose project succeeds in period 1 will continue her project in period 2. But she only has access to a type- L project, which has a negative payoff in period 2. Therefore, the bank can benefit from increasing the period one interest rate R_1 and correspondingly reducing the period two interest rate R_2 given that the type- l borrower whose project succeeds is able to pay in period 1. That is, the bank wants to be paid back as soon as possible when there are type- l borrowers in the primary credit market.

1.2.2 The Secondary Market for Loans

In the secondary market, a bank can sell a loan portfolio backed by its assets. After investors buy the loan portfolios, they implement the same loan contracts offered by banks. Banks want to sell their loans which will generate cash flows in period 2. Their discount fraction δ is lower than the market discount fraction, which is normalized to one.

I further assume that only transactional loans can be traded in the secondary market.⁶ Investors thus buy homogeneous loan portfolios in this market. Hence, knowing the vector μ of the measure of borrowers, investors can correctly predict the credit quality of one share of the loan portfolio. Let V_μ denote the true value of a loan portfolio.

Moreover, assume that the investors are willing to pay $\rho(\gamma)V_\mu$ when the portfolio's value is V_μ , where

$$\rho(\gamma) = \min \{ \max \{ \gamma, \delta \}, 1 \}. \quad (1.6)$$

The price of a loan portfolio depends not only on its credit quality but also on the state of liquidity γ , which is a random variable with support on \mathbf{R}^{++} . That is, if $\gamma \geq 1$, the investors have sufficient liquidity and the portfolio is sold at its true value V_μ ; if $\gamma \in (\delta, 1)$, the investors suffer from the liquidity shortage, and the portfolio is sold at a distressed value γV_μ ; and if $\gamma \leq \delta$, there is no gain from selling the portfolio to investors. I initially assume that the state of liquidity γ is publicly observed.

⁶This is because a bank's effort is unobservable. In practice, institutions like rating agencies can only rate a loan portfolio by observing the (historical) distribution of borrowers in the market, but not a bank's effort to monitor borrowers.

1.2.3 Transactional Lending

Transactional lending uses a passive technology in which the only information required by the bank is the publicly observed first-period outcome. In each period $t = 1, 2$, neither party is obligated to pay anything whenever the project fails. The borrower pays an gross interest rate R_t if the project succeeds. The financial contract is thus a pair of gross interest rates $\{R_1, R_2\}$. Finally, the bank can make a commitment to deny the second-period loan whenever the borrower defaults on the first-period loan.

Borrowers are allocated randomly to banks in the primary credit market. Thus a bank expects the vector μ of the measure of borrowers in its loan portfolio since it cannot prevent the type- l borrowers from borrowing.⁷ Moreover, since the type- l borrowers cannot choose the type- H project, the bank just considers the incentive problem of the type- h borrowers. In this setting, the optimal contract induces the type- h borrowers to choose the type- H project.

Let V_t^1 be a bank's payoff in period $t = 1, 2$ if transactional lending is used. In period 1, the bank's payoff is the same as in equation (1.1): $V_1^1(R_1; \mu) = V_1^0(R_1; \mu)$. The bank's payoff in period 2 is

$$V_2^1(R_2; \mu) = \tilde{\mu}_h v_H(R_2) + \tilde{\mu}_l v_L(R_2). \quad (1.7)$$

The period payoff from the type $\tau \in \{H, L\}$ project satisfies $v_L(R_t) < 0 < v_H(R_t)$ by assumption. But I shall restrict attention to equilibria in which the value of the loan portfolio is positive: $V_t^a(R_t; \mu) > 0$.

Given the state of liquidity γ of the secondary market, the bank maximizes its expected profit,

$$(\text{TL}) \quad \max_{\{R_1, R_2\}} V_1^1(R_1; \mu) + \rho(\gamma) V_2^1(R_2; \mu),$$

subjects to the individual rationality (IR) and incentive compatibility (IC) constraints,

$$(\text{IR}) \quad p_H(\theta - R_1) + p_H^2(\theta - R_2) \geq v_0, \quad (1.8)$$

$$(\text{IC}) \quad p_H(\theta - R_1) + p_H^2(\theta - R_2) \geq p_L(\theta - R_1) + p_L^2(\theta - R_2) + b. \quad (1.9)$$

⁷Recall that a type- l borrower has a positive private benefit $B > v_0$, so she will always apply for a loan regardless of the contract. The bank cannot screen out type- l borrowers. Why? Assume not. Thus there exists a menu of contracts which can separate the two groups of borrowers. But the bank will stop lending to those type- l borrowers who have negative payoff $v_L(R) = p_L R - D < p_L \theta - D < 0$. Then the type- l borrowers will not choose the contract which reveals their type. See Hellwig [31] for a formal argument.

By the IR constraint (1.8), the bank promises at least the reservation utility v_0 to the type- h borrowers. In addition, by the IC constraint (1.9), the type- h borrowers who borrow money from the bank will choose the type- H project given the contract offered. The solution to this problem is presented in Lemma 1.2.

Lemma 1.2 *The optimal transactional loan is as follows:*

1. if the project fails, the gross interest rate in each period $t = 1, 2$ is $R_t^{1*}(0) = 0$;
2. if the project succeeds, the gross interest rates are $R_1^{1*}(\theta) = \theta$ and $R_2^{1*}(\theta) = \theta - v_0/p_H^2$.

To simplify the notation, define $R_1^{1*} \equiv R_1^{1*}(\theta)$ and $R_2^{1*} \equiv R_2^{1*}(\theta)$. The bank maximizes its expected profit by offering the optimal financial contract $\{R_1^{1*}, R_2^{1*}\}$.

To conclude this section, I compare the results of the problems (TL) and (RL) in Figure 1.3. The following results hold for the optimal loan contracts: (1) the interest rates in period 1 are the same $R_1^{1*} = R_1^{0*} = \theta$; and (2) in period 2, the interest rate of relationship loans is larger than that of transactional loans $R_2^{0*} > R_2^{1*}$. The first result is due to the fact that the bank wants to be paid back as soon as possible. The second one comes from the IR constraints: the bank that offers a relationship loan gives the same reservation utility in equilibrium to a type- h borrower. The bank requires a higher interest rate in relationship lending because the type- h borrower has a better chance of continuing her project in relationship lending. The information of the project type has a cost of monitoring the borrower. But this information also relaxes the IC constraint (1.5) in a relationship loan comparing with the IC constraint (1.9) in a transactional loan. Therefore, the bank can increase the second-period interest rate in relationship lending to compensate its cost without violate the IC constraint.

1.3 Decentralized Credit Market

In this section, I relax the assumption that borrowers are assigned to banks randomly in the primary credit market. Now, in the decentralized primary credit market, there exists a continuum of lenders with measure one. Recall that a bank can take an action $a \in \{0, 1\}$, where the actions $a = 0$ and $a = 1$ denote relationship lending and transactional lending,

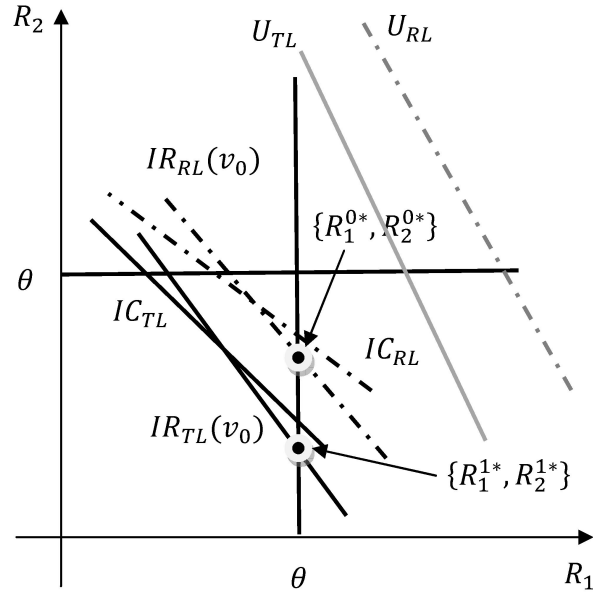


Figure 1.3 Comparing the Optimal Relationship Lending $\{R_1^{0*}, R_2^{0*}\}$ with the Optimal Transactional Lending $\{R_1^{1*}, R_2^{1*}\}$. The R_1 -axis and the R_2 -axis are gross interest rates in the first period and second period, respectively. The pair of optimal relationship lending interest rates $\{R_1^{0*}, R_2^{0*}\}$ is below the incentive compatibility constraint IC_{RL} and lies on the intersecting point of the individual rationality constraint IR_{RL} the limited liability constraint $R_1 = \theta$. Also the pair of optimal transactional lending interest rates $\{R_1^{1*}, R_2^{1*}\}$ is below the incentive compatibility constraint IC_{TL} and lies on the intersecting point of the individual rationality constraint IR_{TL} and the limited liability constraint $R_1 = \theta$. The interest rate R_2^{0*} is higher than R_2^{1*} to compensate the cost to monitor borrowers. See Appendix A for details of the parameters.

respectively. I consider the case in which banks simultaneously and publicly announce their loan technologies. Borrowers then simultaneously choose which technology they prefer. A borrower is then assigned to a random bank that has chosen her preferred technology. If no such bank exists, the borrower does not borrow. I shall prove that the borrowers' searching efforts may change the distribution of borrowers in banks' loan portfolios.

I assume that the fraction of banks that can choose which loans to offer is less than the threshold $\ell_1 < \widehat{\ell}$, where

$$\widehat{\ell} \equiv \frac{(1 - \delta) V_2^1(R_2^{1*}; \mu)}{(1 - \delta) V_2^1(R_2^{1*}; \mu) - [\mu v_L(R_1^{1*}) + \tilde{\mu} v_L(R_2^{1*})]} \in (0, 1). \quad (1.10)$$

Intuitively, for banks that offer transactional loans, the value $(1 - \delta) V_2^1(R_2^{1*}; \mu)$ is the highest potential gain from selling the loan portfolio in the secondary market, and the value $\mu v_L(R_1^{1*}) + \tilde{\mu} v_L(R_2^{1*})$ is the loss caused by the borrowers' searching efforts. This condition requires that the loss is shared among a sufficiently large mass (at least $1 - \widehat{\ell}$) of lenders that offer loans without monitoring. This gives the remaining banks a sufficient incentive to offer transactional loans.

Given the optimal contracts in Lemma 1.2 and Lemma 1.1, type- h borrowers will choose the type- H project, and thus their expected payoff, equal to v_0 , is fixed. In turn, they are indifferent between transactional lending and relationship lending as claimed in the second part of the following Lemma 1.3. It also shows that no type- l borrowers choose banks that offer relationship loans.

Lemma 1.3 *Type- l borrowers strictly prefer to borrow from banks with transactional lending over those with relationship lending, while type- h borrowers are indifferent between the two technologies.*

Proof of Lemma 1.3: The monitoring technology changes the type- l borrower's expected payoff. The amount of private benefit b from a type- L project is exogenously given. Hence, this private benefit of type- l borrowers can be ignored since the technology does not change the probability to receive their loans in period 2. In addition, from the optimal contracts $\{R_1^{0*}, R_2^{0*}\}$ and $\{R_1^{1*}, R_2^{1*}\}$, we know that $R_2^{0*} > R_2^{1*}$ and $R_1^{1*} = R_1^{0*} = \theta$. Therefore, the

following inequality holds,

$$p_L (\theta - R_1^{1*}) + p_L^2 (\theta - R_2^{1*}) + b > p_L (\theta - R_1^{0*}) + p_L^2 (\theta - R_2^{0*}) + b.$$

The left hand side is a type- l borrower's gain from the type- L project if transactional lending is offered; and the right hand side is her gain if relationship lending is offered. This proves the first part of the lemma: in order to have a higher payoff in period 2, the type- l borrowers will look for banks with transactional lending. Q.E.D.

Suppose a proportion $\lambda \in [0, 1]$ of them offer transactional loans. In this case, the measure of lenders that offer loans without monitoring is $\lambda\ell_1 + \ell_0$. Let $\pi_g^a(\lambda)$ represent the mass of type $g \in \{l, h\}$ borrowers in the portfolio of a bank that chooses an action $a \in \{0, 1\}$. Let a vector $\pi^a(\lambda) = (\pi_h^a(\lambda), \pi_l^a(\lambda))$ represent the mass of borrowers. By Lemma 1.3, the type- h borrowers are indifferent between the two loan contracts. Hence the measure is the same as in the representative bank case: $\pi_h^1(\lambda) = \pi_h^0(\lambda) = \mu_h$. But since the type- l borrowers strictly prefer the banks with transactional lending, the measure $\pi_l^1(\lambda)$ is equal to the measure of the type- l borrowers μ_l divided by the measure of banks that offer transactional loans $\lambda\ell_1 + \ell_0$, and the measure $\pi_l^0(\lambda)$ is equal to 0. Summarily, the vectors of the measure of borrowers are $\pi^1(\lambda) = (\mu_h, (\lambda\ell_1 + \ell_0)^{-1} \mu_l)$ and $\pi^0(\lambda) = \pi^0 = (\mu_h, 0)$ for banks that offer transactional lending and relationship lending, respectively. Finally, let $\tilde{\pi}_l^1(\lambda) = \pi_l^1(\lambda) p_L$ be the mass of borrowers whose projects succeed.

1.3.1 Interbank Relationships

Apply the vectors of the measure of borrowers $\pi^a(\lambda)$ for $a \in \{0, 1\}$ and the results from the optimization problems (TL) and (RL) to compute a bank's profit. Each bank $i \in [0, \ell_1]$ has a same profit function $U : \{0, 1\} \times [0, 1] \times \mathbf{R}^{++} \rightarrow \mathbf{R}^+$, where $U(a, \lambda, \gamma)$ is the bank's profit if it chooses an action a , a proportion λ of banks offer transactional loans, and the state of liquidity is γ . Specifically, if the bank chooses transactional lending, the profit is

$$U(1, \lambda, \gamma) = V_1^1(R_1^{1*}; \pi^1(\lambda)) + \rho(\gamma) V_2^1(R_2^{1*}; \tilde{\pi}_l^1(\lambda)), \quad (1.11)$$

the period 1 payoff with interest rate R_1^{1*} , plus $\rho(\gamma)$ times the period 2 payoff with interest rate R_2^{1*} ; and if the bank chooses relationship lending, the profit is

$$U(0, \lambda, \gamma) = V_1^0(R_1^{0*}; \pi^0) - C(m, \pi^0) + \delta V_2^0(R_2^{0*}; \pi^0), \quad (1.12)$$

the period 1 payoff with interest rate R_1^{0*} , less the monitoring cost, plus the discount rate δ times the period 2 payoff with interest rate R_2^{0*} . Noticing in the latter case the bank's profit depends on neither the proportion λ nor the state of liquidity γ , let us denote $U(0, \lambda, \gamma) \equiv U_0$.

To analyze a bank's best response, it is enough to know the profit gain from choosing one action rather than the other. The profit gain from choosing transactional lending over relationship lending is parameterized by a function $\omega : [0, 1] \times \mathbf{R}^{++} \rightarrow \mathbf{R}$ with $\omega(\lambda, \gamma) = U(1, \lambda, \gamma) - U_0$. From equations (1.11) and (1.12), the profit gain is given by

$$\omega(\lambda, \gamma) = V_1^1(R_1^{1*}; \pi^1(\lambda)) + \rho(\gamma) V_2^1(R_2^{1*}; \pi^1(\lambda)) - U_0. \quad (1.13)$$

At the heart of this model is a set of properties (B1-B3) on the profit gain $\omega(\lambda, \gamma)$. I discuss the intuition of these properties in this section and the proves are in Appendix A.

- B1. State Monotonicity.** The profit gain $\omega(\lambda, \gamma)$ is non-decreasing in the state of liquidity γ .
- B2. Strategic Complementarities.** The profit gain $\omega(\lambda, \gamma)$ is non-decreasing in the proportion λ of banks that offer transactional loans.
- B3. Dominance Regions.** There exist the upper and lower bounds $\underline{\gamma}, \bar{\gamma} \in (\delta, 1)$ such that: (1) the profit gain is negative $\omega(\lambda, \gamma) < 0$ for all the proportion λ and the state of liquidity $\gamma < \underline{\gamma}$; and (2) the profit gain is positive $\omega(\lambda, \gamma) > 0$ for all the proportion λ and the state of liquidity $\gamma > \bar{\gamma}$.

I assume that there are regions of extremely good and bad states of liquidity in which a bank's best response is independent of its belief concerning the responses of others. That is, when the state of liquidity is extremely bad $\gamma \leq \underline{\gamma}$, the expected profit from transactional lending is always lower than relationship lending. A bank's best response is to offer relationship lending. Similarly, when the state of liquidity is extremely good $\gamma \geq \bar{\gamma}$, the expected profit

from transactional lending is always higher than relationship lending. A bank's best response is to use transactional lending.

First, I want to verify the state monotonicity property. Intuitively, if the state of liquidity γ is in the interval $[\delta, 1)$, once there is a small positive liquidity shock, the investors will pay more for the loan portfolios with value $V_2^1(R_2^{1*}; \pi^1(\lambda))$. Banks using transactional lending benefit since they can sell their loan portfolios at this higher price. Note that banks will retain their loans if the state of liquidity is in $(0, \delta)$. In addition, investors have sufficient liquidity for loan portfolios if the state of liquidity is in $[1, \infty)$. In turn, if the state of liquidity γ is not in $[\delta, 1)$ then a small positive liquidity shock will not change the profit of banks that offer transactional lending. Moreover, banks using relationship lending are not affected by any shock since they retain their loans. Therefore, a bank's relative profit from choosing the transactional technology is non-decreasing in the state of liquidity γ .

As for strategic complementarities, when more banks offer transactional lending, the profit of each such bank rises because the measure of the type- L projects $\pi_l^1(\lambda)$ in the portfolio decreases. But there is no effect on the profit of a bank that chooses the relational technology since the vector of the measure of borrowers π^0 of each group stays the same. Hence, the profit gain $\omega(\lambda, \gamma)$ from choosing transactional lending over relationship lending is increasing in the proportion λ .

Finally, the assumption of dominance regions requires a modest individual monitoring cost. Specifically, define the lower bound \underline{m} and the upper bound \overline{m} of the individual monitoring cost

$$\underline{m} \equiv \frac{\delta\mu_h v_H(R_2^{0*}) - \ell_0^{-1}\mu_l v_L(R_1^{1*}) - V_2^1(R_2^{1*}; \pi^1(0))}{\mu_h(1-p_H)}, \quad (1.14)$$

$$\overline{m} \equiv \frac{\delta\mu_h v_H(R_2^{0*}) - \mu_l v_L(R_1^{1*}) - \delta V_2^1(R_2^{1*}; \mu)}{\mu_h(1-p_H)}. \quad (1.15)$$

When the individual cost m is less than the upper bound \overline{m} , the total cost $\mu_h(1-p_H)m$ is small enough that an individual bank has an incentive to offer relationship lending even no other banks do the same $\lambda = 1$ once the state of liquidity γ is in $[0, \delta]$. Similarly, when the individual cost m is larger than the lower bound \underline{m} , the total cost $\mu_h(1-p_H)m$ is large enough that an individual bank has an incentive to offer transactional lending even no other

banks indexed by $i \in [0, \ell_1]$ do the same $\lambda = 0$ once the state of liquidity is in $[1, \infty)$. In order to ensure the existence of dominance regions, I assume the monitoring cost m is in the interval (\underline{m}, \bar{m}) .⁸

1.3.2 Multiple Equilibria

Initially, assume that the liquidity parameter λ is commonly observed. Let us restrict attention to pure strategy symmetric Nash equilibria. Formally, a (pure) strategy of all banks $i \in [0, \ell_1]$ is a function $s : \mathbf{R}^{++} \rightarrow \{0, 1\}$, where $s(\gamma)$ is the chosen action when the state of liquidity is γ . The strategy consists of a decision to offer relationship loans or transactional loans.

The optimal loan contract is exogenously given once the banks choose which technology to use. The proportion λ of banks that offer transactional lending is in the set $\{0, 1\}$ due to symmetry. In addition, this proportion is perfectly predicted by the banks in equilibrium. Therefore, the vectors of the measure of borrowers $\pi^a(\lambda)$ for $a \in \{0, 1\}$ are known by the banks.

Given a sufficiently large mass of banks that always offer transactional loans, and given the individual monitoring cost $m \in (\underline{m}, \bar{m})$, the model has multiple equilibria. Define a lower bound $\underline{\gamma}$ and an upper bound $\bar{\gamma}$ of the state of liquidity:

$$\underline{\gamma} \equiv \frac{\delta \mu_h v_H (R_2^{0*}) - \mu_l v_L (R_1^{1*}) - \mu_h (1 - p_H) m}{V_2^1 (R_2^{1*}; \mu)}, \quad (1.16)$$

$$\bar{\gamma} \equiv \frac{\delta \mu_h v_H (R_2^{0*}) - \ell_0^{-1} \mu_l v_L (R_1^{1*}) - \mu_h (1 - p_H) m}{V_2^1 (R_2^{1*}; \pi(1, 0))}. \quad (1.17)$$

It is easy to see that $\delta < \underline{\gamma} < \bar{\gamma} < 1$ from the property **B3**. Thus I have $\rho(\gamma) = \gamma$ by (1.6).

Theorem 1.4 *When $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$, there exist two (pure strategy symmetric) equilibria: (1) all banks offer transactional lending, or (2) all banks $i \in [0, \ell_1]$ offer relationship lending.*

Proof of Theorem 1.4: First, assume all other banks offer transactional lending, $\lambda = 1$. When the state of liquidity $\gamma = \underline{\gamma}$, the profit gain is non-negative, $\omega(1, \underline{\gamma}) \geq 0$. By the state monotonicity (B1), I have $\omega(1, \gamma) \geq 0$ for all states of liquidity $\gamma \geq \underline{\gamma}$. That is, no individual bank has an incentive to offer relationship lending. Hence, there is an equilibrium in which

⁸It is easy to check that $\underline{m} < \bar{m}$ when $\ell_1 \leq \hat{\ell}$. See Appendix A.

all banks offer transactional lending. Now, assume all other banks $i \in [0, \ell_1]$ offer relationship lending, $\lambda = 0$. When $\gamma = \bar{\gamma}$, the profit gain is non-positive $\omega(0, \bar{\gamma}) \leq 0$. Again, by state monotonicity, I have $\omega(0, \gamma) \leq 0$ for all $\gamma \leq \bar{\gamma}$. That is, no individual bank $i \in [0, \ell_1]$ has an incentive to offer transactional lending. There is a second equilibrium in which all banks $i \in [0, \ell_1]$ offer relationship lending. Q.E.D.

1.4 Private Signals: Unique Equilibrium

In the previous section, I have shown that when the state of liquidity is publicly observed, there exist multiple equilibria. However, when the model has multiple equilibria, it is hard to predict which equilibrium will occur. The global game approach (see Carlsson and van Damme [12]; Morris and Shin [47]) provides a natural way to address the problem. To use this approach, I modify the model. The state of liquidity of the secondary market is no longer publicly observed. Instead, each bank receives a slight noisy private signal regarding the state of liquidity in period 1. The signals can be thought of as private information or private opinion regarding the investors' state of liquidity. The introduction of private signals changes the results considerably.

Suppose that the state of liquidity γ has a log-normal distribution F (density f). Each bank $i \in [0, \ell_1]$ observes its private signal $x_i = \gamma \exp(\sigma \eta_i)$, where $\sigma > 0$ is a scale factor and η_i (the noises) are independent random variables, each with a standard normal distribution Φ (density ϕ). The signals are used to coordinate the banks' actions.

I denote by $\Gamma(\sigma)$ this incomplete information game and consider a pure strategy Perfect Bayesian Equilibrium. Similar to the public signal case, a (pure) strategy of bank i in this private signal case is a function $s_i : \mathbf{R}^{++} \rightarrow \{0, 1\}$, where $s_i(x)$ is the action chosen if the bank observes its private signal x . A strategy profile is $s = (s_i)_{i \in [0, \ell_1]}$.

1.4.1 Solving the Model

In a general model, Frankel, Morris, and Pauzner [24] proved, as the signal noise vanishes, a unique strategy profile survives iterative dominance. This model fits their setting: a continuum of players and two actions.

Consider a bank that has observed a signal x and knows that all other banks indexed by $i \in [0, \ell_1]$ will offer relationship lending if they observe signals less than y . Let $\omega_\sigma(x, y)$ denote the bank's expected profit gain from choosing transactional lending over relationship lending. The main result predicts a unique equilibrium. The proof is in Appendix A.

Theorem 1.5 *The game $\Gamma(\sigma)$ essentially has a unique equilibrium in which banks indexed by $i \in [0, \ell_1]$ offer transactional lending if they observe a signal above the threshold x^* (i.e., $s_i(x) = 1$ for all $x > x^*$) and relationship lending if below (i.e., $s_i(x) = 0$ for all $x < x^*$), where the threshold $x^* \in (\underline{\gamma}, \bar{\gamma})$ is determined by the equation*

$$\omega_\sigma^*(x, x) = \int_{\lambda=0}^1 \omega(\lambda, x) d\lambda = 0. \quad (1.18)$$

It is well known that the equilibrium strategies and beliefs of the two action model do not depend on the structure of the noise as the noise vanishes. Morris and Shin [47] offer an explanation based on the contagion argument. The banks' actions are determined by their signals: they use transactional lending if and only if their signals are below the threshold x^* determined by the equation (1.18). Theorem 1.5 thus provides the method to compute the threshold x^* . From the equation (1.18) and the definition (1.13), the equality $\omega_\sigma^*(x, x) = 0$ implies $\rho(x^*)X_1 + X_2 = 0$, where

$$\begin{aligned} X_1 &= \tilde{\mu}_h v_H(R_2^{1*}) + (\ell_1^{-1} \ln \ell_0^{-1}) \tilde{\mu}_l v_L(R_2^{1*}) > 0, \text{ and} \\ X_2 &= (\ell_1^{-1} \ln \ell_0^{-1}) \mu_l v_L(R_1^{1*}) + \mu_h [(1 - p_H) m - \delta v_H(R_2^{0*})] < 0. \end{aligned}$$

Since the threshold x^* is in the interval $(\underline{\gamma}, \bar{\gamma})$, I have $\rho(x^*) = x^*$ by the definition (1.6). Hence, the threshold is

$$x^* = -X_2/X_1. \quad (1.19)$$

To conclude this section, I compare banks' behavior in this full model with that in the benchmark case. The gain from choosing transactional lending over relationship lending in the benchmark case is

$$\begin{aligned} \omega(\gamma) &= \mu_l v_L(R_1^{1*}) + \rho(\gamma) [\tilde{\mu}_h v_H(R_2^{1*}) + \mu_l p_L v_L(R_2^{1*})] \\ &\quad + \mu_h [(1 - p_H) m - \delta v_H(R_2^{0*})]. \end{aligned}$$

Hence, the bank will choose transactional lending if $\omega^*(\gamma) > 0$. By Theorem 1.4 the threshold γ^* is in the interval $(\underline{\gamma}, \bar{\gamma})$ and $\rho(\gamma^*) = \gamma^*$. Therefore, I have the threshold

$$\gamma^* = -\frac{\mu_l v_L (R_1^{1*}) + \mu_h [(1 - p_H) m - \delta v_H (R_2^{0*})]}{\tilde{\mu}_h v_H (R_2^{1*}) + \tilde{\mu}_l v_L (R_2^{1*})}.$$

It is easy to check that $\gamma^* < x^*$.⁹

Intuitively, when the bank is uncertain which type of loans its opponents will offer, it will take a more cautious action - offering transactional lending when the observed state of liquidity is higher. This may increase the monitoring cost of banks. However, if monitoring reduces the proportion of lower quality borrowers and hence the economy-wide default risk, the uncertainty may increase the welfare.

1.4.2 The Loan Market Analysis

In this section, I analyze the secondary market for loans. The threshold x^* is determined by equation (1.19). Assume the realized state of liquidity of the secondary market is $\hat{\gamma}$. Knowing the realization of the state of liquidity, I can calculate exactly the proportion of banks that offer transactional lending. Given a small scale $\sigma > 0$,¹⁰ the noises are $\sigma \eta_i$ with η_i independent and standard normal for all $i \in [0, \ell_1]$. By the law of large numbers, the fraction $\hat{\lambda}$ of banks that offer transactional lending is determined by

$$\hat{\lambda} = 1 - \Phi\left(\frac{1}{\sigma} \ln(x^*/\hat{\gamma})\right), \quad (1.20)$$

and the rest of banks will offer relationship lending.

The effect of an increasing in the individual monitoring cost to the fraction of banks in the secondary market is summarized below.

Proposition 1.6 *When the individual monitoring cost m increases, the fraction $\hat{\lambda}$ of banks that offer transactional lending increases. In addition, when the realized state of liquidity $\hat{\gamma}$ increases, the fraction $\hat{\lambda}$ of banks that offer transactional lending increases.*

⁹When $\ell_1 \in (0, 1)$, the following inequality holds $\ell_1^{-1} \ln \ell_0^{-1} > 1$.

¹⁰See Morris and Shin [48] for formal treatment that, when the scale σ is small related to the distribution of the fundamentals, the threshold x^* is also determined by (1.18).

Proof of Proposition 1.6: Let us only consider the individual monitoring cost m in the interior of the interval $(\underline{m}, \overline{m})$, and the proof is intuitive. Taking a derivative of the equation (1.20), I have

$$\frac{\partial \hat{\lambda}}{\partial m} = \frac{\partial \hat{\lambda}}{\partial x^*} \frac{\partial x^*}{\partial m} = -\frac{\mu_h (1 - p_H)}{\sigma X_2} \phi \left(\frac{1}{\sigma} \ln (x^* / \hat{\gamma}) \right) > 0.$$

Therefore, the fraction $\hat{\lambda}$ is an increasing function of the individual cost m .

The fraction $\hat{\lambda}$ of banks that offer transactional lending is an increasing function of the realized state of liquidity $\hat{\gamma}$:

$$\frac{\partial \hat{\lambda}}{\partial \hat{\gamma}} = \frac{1}{\sigma \hat{\gamma}} \phi \left(\frac{1}{\sigma} \ln (x^* / \hat{\gamma}) \right) > 0.$$

Therefore, when the investors have a relaxed liquidity constraint, the secondary market will experience a boom. Q.E.D.

Figure 1.4 simulates the function (1.20) and also shows the effects of liquidity shocks and changes in the individual monitoring cost. In this model, the monitoring cost is a cost to learn the true type of project. In practice, the technological innovation reduces the cost of information storage or the cost of transportation. However, the development of the multinational corporations and the appearance of internet companies (e-business) have made it harder to build relationships to monitor the firms closely (see Petersen and Rajan [53]; Degryse and Ongena [14]). The recent fast growth of the secondary market for transactional loans may reflect such a cost increasing in the primary credit market (from A to A' in Figure 1.4).

Proposition 1.7 *With a change in the realized state of liquidity $\hat{\gamma}$, the model has two predictions on the secondary market for loans:*

1. *the quality of individual loan portfolio of each bank increases (decreases) when the state of liquidity becomes better (or worse, respectively); and*
2. *the fraction of the type- l borrowers in the credit market increases (decreases) when the state of liquidity becomes better (or worse, respectively).*

Proof of Proposition 1.7: First, in the secondary market, for each bank that offers transactional lending, it faces the mass $(\hat{\lambda} \ell_1 + \ell_0)^{-1} \tilde{\mu}_l$ of the type- l borrowers with the type- L

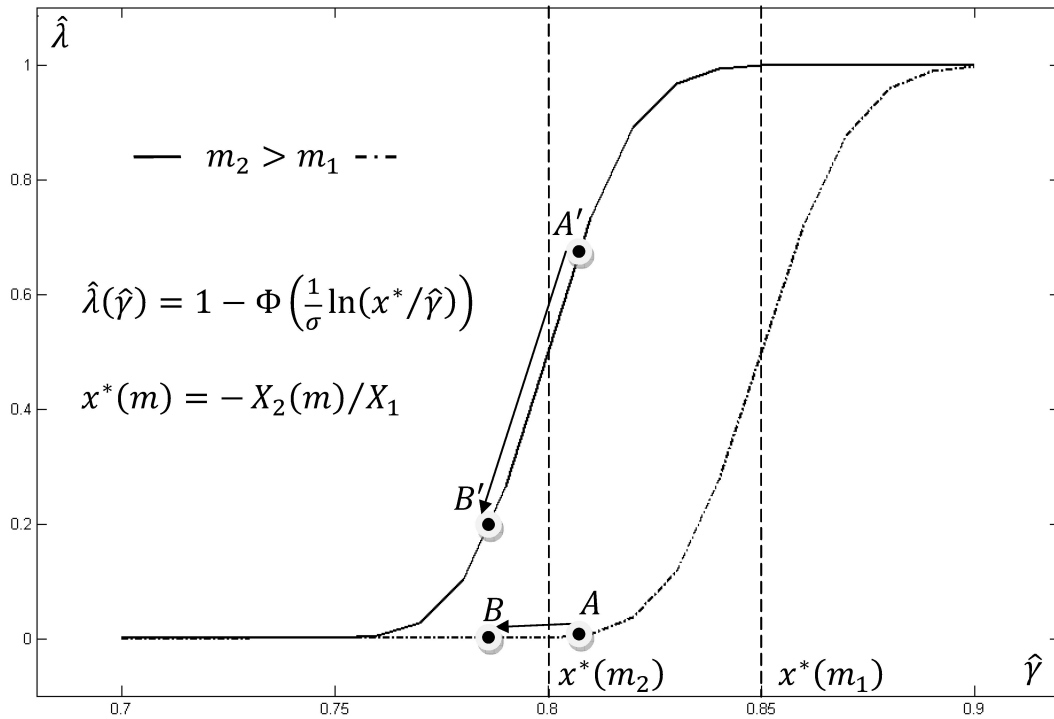


Figure 1.4 The Fraction of Banks Offering Transactional Loans. The horizontal axis $\hat{\gamma}$ is the state of liquidity and the vertical axis $\hat{\lambda}$ is the fraction of banks who offer transactional loans. When the signals are precise (or the noises are small), the liquidity shock has a very sharp effect on the fraction around the threshold. For example, in the figure, the threshold is $x_1^* = 0.8$ and the scale of noise is $\sigma = 0.02$, a small drop of the realized state of liquidity will reduce the fraction of banks off transactional loans significantly.

project. But the measure of the type- h borrowers with the type- H project in each bank is $\tilde{\mu}_h$, which is independent of the fraction $\hat{\lambda}$. By the equation (1.20), after an increasing in the state of liquidity $\hat{\gamma}$, the proportion of the type- l borrowers decrease in each loan portfolio. In other words, the quality of loan portfolios increases when the state of liquidity becomes better.

To see the second prediction, notice that the total measure of the type- l borrowers in the credit market is fixed. The banks offer relationship loans find out the true types of projects and continue the loans to the good projects. On the other hand, from the discussion in Section 1.2, banks offering transactional loans exert less efforts to monitor borrowers. Therefore, in period 2, the type- h borrowers with good projects are driven out of the primary credit market more often when the banks offer transactional loans. Q.E.D.

Hence, when the state of liquidity $\hat{\gamma}$ becomes better, the default risk of the economy could increase sharply, which may reduce the social welfare. Inefficiency may be imbedded in the development of the secondary market since “hot potatoes” (bad loans) sit in the financial system. When the state of liquidity $\hat{\gamma}$ becomes worse, banks will switch back to the safe but costly relationship lending (from A' to B' in Figure 1.4).

1.5 Conclusion

In this chapter, I study a lending model and consider the endogeneity of lenders' information structures. I assume that each bank can decide whether or not to discover the quality of a borrower's project to use the active or passive technology. Information acquisition (the active technology) is productive because it can find out the good borrowers with a given cost.

In addition, I study the case in which liquidity shocks in the secondary market for transactional loans change the relative payoff to different loan technologies and thus the exogenous shocks may alter a bank's incentive to acquire information. Given that banks have slightly noisy private signals of the liquidity shocks, the model has a unique equilibrium.

In this two-period model, borrowers belong to different types and they choose the types of their projects in the first period. The types of the borrowers and the types of the projects are private information of borrowers and are perfectly correlated in the first and second period.

So this model is a false dynamic model (Laffont and Martimort [41], p.269). The structure is simplified in order to have an explicit solution when I add endogenous information acquisition and liquidity shocks. But it might be interesting to study the effects of adding repeated periods.

The secondary loan market is important source of financial instability. In this model, the strategic complementarities come from the borrowers' search efforts. In addition, I introduce the liquidity shocks to investors' willingness to pay in the secondary market. When the investors' willingness to pay decreases, the payoff of the banks that offer transactional lending decreases. The aggregate value traded in the secondary market drops even faster because fewer banks are going to offer transactional loans, which worsens the distribution of borrowers in the loan portfolios of the transactional lenders.

CHAPTER 2. SECURITIZATION AND LENDING COMPETITION

by David M. Frankel and Yu Jin

2.1 Introduction

Securitization of conventional home mortgages began in 1970 with the founding of the Federal Home Loan Mortgage Corporation.¹ The proportion of mortgages held in market-based instruments rose steadily from 20% in 1980 to 68% in 2008.² Earlier evidence indicates that securitization has been growing at least since 1975 (Jaffee and Rosen [37, Table 2]).

Remote lending has also grown. Petersen and Rajan [53, Figures I and II] find an upwards trend in distances between small firms and their lenders that began in about 1978 or 1979 and continued through the end of their data in 1992. The mean borrower-lender distance in a sample of small business loans studied by De Young, Glennon, and Nigro [19, pp. 125-6] rose from 5.9 miles in 1984 to 21.5 miles in 2001. Remote lending of residential mortgages also rose from 1992 to 2007 (Loutskina and Strahan [43, p. 1477], discussed below).

We present a tractable theoretical model that links securitization and remote lending. We assume that banks have hard information about all loan applicants but soft information about only local applicants. Without securitization, banks lend only to local applicants because of a winner's curse. With securitization, in contrast, ignorance is bliss: the less a bank knows about its loans, the less of a lemons problem it faces in selling them.³ This enables banks to compete successfully for some remote applicants.

Our model yields many predictions that are consistent with prior empirical findings (section

¹A detailed history of securitization appears in Hill [33].

²The source is unpublished data underlying Figure 3 in Shin [59].

³In a prior empirical paper, Loutskina and Strahan [43] point out that banks may have an incentive to lend remotely in order to avoid private information at the time of securitization.

2.5.1):

1. **Securitization Stimulates Lending.** As in Shin [59], securitization leads to expanded lending by connecting liquid investors with loan applicants. There is considerable evidence that the securitization boom in the 2000s led to expanded lending (Demyanyk and Van Hemert [18]; Krainer and Laderman [40]; Mian and Sufi [44]).
2. **Securitization Favors Remote Lending.** In our model, banks lend remotely only if they can securitize their loans. Moreover, a bank securitizes all of its remote loans but only some of its local loans. Loutskina and Strahan [43] find that as securitization rose, the market share of concentrated lenders - those which originate at least 75% of their mortgages in one MSA - fell from 20% to 4% from 1992 to 2007. Moreover, concentrated lenders retain a higher proportion of their loans. Finally, when they expand to new MSA's, these lenders are more likely to sell their remote loans than those made in their core MSA's.
3. **Remote Borrowers have Strong Observables but High Conditional Default Rates.** While a bank might lend to a local applicant who has a low credit score in our model, it will not do so for a remote one whose credit score is all it sees. Hence, remote borrowers tend to have stronger observables than local borrowers. (We use "borrower" to refer to an applicant who gets a loan.) On the other hand, since banks lack soft information for remote applicants, they make worse lending decisions: conditional on observables, distant borrowers are more likely to default.⁴ Loutskina and Strahan [43, p. 1456] find that concentrated lenders (defined above) have lower loan losses despite lending to applicants who are riskier in terms of loan to value ratios. Agarwal and Hauswald [1] find that applicants with strong observables tend to apply online for loans, while in-person applicants tend to be those with weaker observables but positive estimates of the bank's soft information about them. Moreover, online loans default more than observationally equivalent in-person loans. De Young, Glennon, and Nigro [19] find that banks that lend

⁴This empirical implication is also present in the prior theoretical model of Hauswald and Marquez [30], which we discuss in section 2.6.3.

remotely have higher default rates.

4. **Securitization Lets Borrowers with Strong Observables Get Cheap Remote**

Loans. In our model, securitization encourages banks to lend to remote applicants with strong observables. They must offer low interest rates to these applicants in order to prevent cream skimming by the applicants' local banks. In contrast, banks can demand high interest rates from quality local applicants whose observables are weak since these applicants cannot get remote loans. This has two empirical implications. First, the securitization boom in the 2000s should have strengthened the (negative) relation between borrower observables and interest rates. Rajan, Seru, and Vig [57] find that borrower credit scores and LTV ratios explain just 9% of interest rate variation among loans originated in 1997-2000 but 46% of this variation among loans originated in 2006. A second implication is that remote borrowers pay lower rates.⁵ Agarwal and Hauswald [1] find that internet loans carry lower interest rates than in-person loans. Degryse and Ongena [14] find that interest rates decrease with the distance between small firms and their lenders in Belgium. Mistrulli and Casolaro [46] find the same relation among business lines of credit in Italy.

5. **Securitization Raises Conditional and Unconditional Default Rates.**

Securitization encourages more remote lending in our model. This raises default rates conditional on borrower observables. Securitization also makes lending more profitable in general, which encourages banks to lower lending standards as in Shin [59]. For both reasons, the unconditional default rate also rises. These predictions are confirmed by empirical research. Rajan, Seru, and Vig [57] find that conditional default rates rose between 1997-2000 and 2001-6.⁶ Demyanyk and Van Hemert [18] find that conditional and unconditional default rates rose from 2001 to 2007.⁷

⁵The comment in footnote 4 applies here as well.

⁶They control for the loan interest rate, credit score, loan to value ratio, and dummy variables for adjustable rates, prepayment penalties, and whether the lender lacked documentation of the borrower's income or assets.

⁷Their controls include the loan interest rate, borrower credit score, loan to value ratio, debt to income ratio, local changes in house prices and unemployment since origination, and dummies for prepayment penalties, owner-occupier status, and low documentation.

6. Securitized Loans Have Higher Conditional Default Rates than Retained

Loans. In our model, local banks adopt lower lending standards in local areas that are more profitable to securitize. Hence, securitized loans have higher default rates than retained loans conditional on observables. Krainer and Laderman [40] find that controlling for observables, privately securitized loans default at a higher rate than retained loans. Elul [23] finds that securitized loans perform worse than observationally similar unsecuritized loans, and that the effect is strongest in the prime market.

In our model, securitization has mixed effects on social welfare. It raises the supply of funding for worthwhile projects by connecting liquid investors with deserving loan applicants. However, it also leads to an inefficient loan allocation by giving banks an incentive to favor remote applicants with strong observables. For instance, consider two applicants in the same location. One has a high credit score but a negative NPV project. The other has a low credit score but a positive NPV project. A remote bank would favor the first applicant since evaluating a project's NPV requires soft information, which it lacks. A local bank may prefer not to fund either applicant because it knows too much about them, which makes their loans difficult to sell. Hence, funds go to the negative-NPV project, which is clearly inefficient.

We treat securitization as an exogenous innovation that encourages remote lending. If instead securitization were initially possible and an exogenous barrier to remote lending were then lifted, our model would also predict a simultaneous increase in both remote lending and securitization.⁸ In practice, legal barriers to interstate banking fell gradually starting in Maine in 1978 and ending with the federal government's passage of the Interstate Banking and Branching Efficiency Act of 1994, which abolished all remaining restrictions (Loutskina and Strahan [43, pp. 1451-2]). Since securitization was invented earlier, these barriers may have fallen partly in response to pressure from large banks who were eager to increase their securitization profits. Alternatively, their fall may have been due to an exogenous change in regulatory philosophy. This is an interesting topic for future empirical research.

The rest of the chapter is as follows. The model is presented in section 2.2. Section 2.3

⁸Since we assume banks lack private information about their remote loans and have a lower discount factor than investors, banks securitize all of their remote loans. Since - in our model - they securitize only some of their local loans, removing a barrier to remote lending would raise the proportion of loans that are securitized.

analyzes a base case without securitization, while the full model is studied in section 2.4. The model's predictions are discussed and illustrated in section 2.5. Section 2.6 reviews related theoretical literature, while conclusions appear in section 2.7.

2.2 The Model

A country consist of two *ex ante* identical regions, A and B , each containing a single bank. We will refer to the bank in region A (B) as bank a (respectively, b). Each region $R \in \{A, B\}$ consists of a continuum of locations $\ell \in [0, 1]$. In each location ℓ there is a continuum of agents. All participants are risk-neutral.

Each agent has a project that requires one unit of capital and pays a fixed gross return of $\rho > 1$ if it succeeds and zero otherwise. The project's success probability is the product of the agent's unknown type $\theta \in (0, 1)$ and a macroeconomic shock $L_\ell^R \in (0, 1)$ to the agent's location ℓ in the region R in which she lives. Project outcomes, conditional on these success probabilities, are independent.⁹

There are four periods, $t = 1, 2, 3, 4$. Period 1 is the lending stage. The banks see signals of each agent's type θ and then make competing loan offers to the agents. This stage determines which agents borrow from which banks, and at what interest rates. Period 2 is the security design stage. Each bank decides which loans to securitize and what liquidating dividend to pay as a function of the returns of these loans. Period 3 is the signalling stage. The bank in each region R first sees signals of its local macroeconomic shocks L_ℓ^R . Each bank then chooses how many shares of its security to sell to investors. Period 4 is the settlement stage: project returns are realized, successful borrowers repay their loans, and each bank pays a liquidating dividend to holders of its security.

The local shock L_ℓ^R has the form

$$L_\ell^R = \sum_{k=1}^K \alpha_{k\ell}^R \zeta_k^R. \quad (2.1)$$

For each k , $\zeta_k^R \in (0, 1)$ is a random variable that is realized after the security is sold and $\alpha_{k\ell}^R \in [0, 1]$ is a constant satisfying $\sum_{k=1}^K \alpha_{k\ell}^R \leq 1$.¹⁰ We refer to ζ_k^R as the k th local factor

⁹That is, a project's success probability is θS_ℓ^R regardless of the outcomes of other projects.

¹⁰One can include a constant term in equation (2.1) by assuming that one of the factors is a constant.

in region R and to $\alpha_{k\ell}^R$ as location ℓ 's loading on this factor. For instance, each factor may represent an industry and the factor loading may be the share of a location's workforce that is employed in the industry.¹¹ In each region R , the distribution of the factor loading vector $(\alpha_{k\ell}^R)_{k=1}^K$ across locations $\ell \in [0, 1]$ has no atoms.¹²

At the beginning of period 1, both banks see a public signal $s_{\text{pub}} \in (0, 1)$ of type θ of each agent. Simultaneously, the agent's local bank also sees a private signal $s_{\text{priv}} \in (0, 1)$ of θ .¹³ The joint population distribution of the type θ , signals s_{pub} and s_{priv} , and location ℓ is given by a known distribution function F and associated continuous density function f on the domain $(0, 1)^3 \times [0, 1]$.

The assumption that F is region-independent is purely for notational convenience. It could be replaced by region-specific distribution functions F^A and F^B with no change in the results, except for the proliferation of region superscripts throughout the chapter. The same is true of all distributions derived from F . In particular, we will also use F to denote the marginal and conditional distribution functions of these variables or subsets of them; for instance, $F(\theta|s_{\text{priv}}, s_{\text{pub}}, \ell)$ denotes the conditional distribution of θ given s_{priv} , s_{pub} , and ℓ . The corresponding densities are written with “ f ” in place of “ F ”, and we assume that all such densities are continuous.

We assume that an increase in the public signal - or in the private signal *conditional* on the public signal - raises the conditional distribution of θ in a first-order stochastic dominance sense. This is formalized in the following two assumptions. The first says that an increase in the public signal weakly lowers the probability of observing a type θ below any given threshold, and strictly lowers the average of these probabilities across thresholds. Moreover, this effect is bounded above. The second property is like the first but relates to the effect of the private signal on the distribution of types conditional on the public signal. (In both cases, we also condition this distribution on the location ℓ .)

Public Signal Monotonicity For any signal $s_{\text{pub}} \in (0, 1)$ and location $\ell \in [0, 1]$, there are

¹¹Factor dependence within and across regions is permitted, as detailed below in section 2.2.1.3.

¹²That is, there is no factor loading vector that receives a strictly positive probability weight.

¹³The outcome of the model will not depend on what the applicant knows about her own type, as the applicant simply borrows from the bank that offers her the lower interest rate.

integrable functions $\underline{\lambda} \leq \bar{\lambda} : (0, 1) \rightarrow \mathfrak{R}_+$, such that the integral $\int_{\theta=0}^1 \underline{\lambda}(\theta) d\theta$ is strictly positive and for each $\theta \in (0, 1)$, the derivative $\frac{\partial F(\theta|s_{\text{pub}}, \ell)}{\partial s_{\text{pub}}}$ exists and lies between $-\bar{\lambda}(\theta)$ and $-\underline{\lambda}(\theta)$, inclusive.

Private Signal Monotonicity For any signals $s_{\text{pub}}, s_{\text{priv}} \in (0, 1)$ and location $\ell \in [0, 1]$, there are integrable functions $\underline{\mu} \leq \bar{\mu} : (0, 1) \rightarrow \mathfrak{R}_+$ such that the integral $\int_{\theta=0}^1 \underline{\mu}(\theta) d\theta$ is strictly positive and for each $\theta \in (0, 1)$, the derivative $\frac{\partial F(\theta|s_{\text{priv}}, s_{\text{pub}}, \ell)}{\partial s_{\text{priv}}}$ exists and lies between $-\bar{\mu}(\theta)$ and $-\underline{\mu}(\theta)$, inclusive.

Let $\eta = E[\theta|s_{\text{pub}}, \ell] \stackrel{d}{=} \eta(s_{\text{pub}}|\ell)$ denote an agent's expected type given her public signal and location; let $\nu = \eta^{-1} E[\theta|s_{\text{pub}}, s_{\text{priv}}, \ell] \stackrel{d}{=} \nu(s_{\text{priv}}|s_{\text{pub}}, \ell)$ denote the proportional change in this expectation that results from learning her local bank's private signal.¹⁴ By the Law of Iterated Expectations, $E(\nu|\eta, \ell)$ is identically equal to one.

Henceforth, we will work directly with η and ν , which we refer to respectively as the agent's *credit score* and *private type*. The following result states that (a) the credit score is strictly increasing in the public signal and (b) conditional on the public signal, the private type is strictly increasing in the private signal. Moreover, both rates of increase are bounded.

Lemma 2.1 *The functions $\eta(s_{\text{pub}}|\ell)$ and $\nu(s_{\text{priv}}|s_{\text{pub}}, \ell)$ have slopes (with respect to s_{pub} and s_{priv} , respectively) that are strictly positive and finite.*

Lemma 2.1 has the following useful implication. Let us say the pair (η, ℓ) is *feasible* if the location ℓ is in $[0, 1]$ and the credit score η lies strictly between $\sup_{s_{\text{pub}}} \eta(s_{\text{pub}}|\ell)$ and $\inf_{s_{\text{pub}}} \eta(s_{\text{pub}}|\ell)$. All feasible pairs have a finite, strictly positive probability density:

Lemma 2.2 *The pair (η, ℓ) is distributed according to a finite density g which is strictly positive on the set of feasible pairs (η, ℓ) .*

Let the distribution function of (η, ℓ) be denoted $G(\eta, \nu)$. Let the conditional distribution function of the private type ν given the credit score η and location ℓ be denoted $H(\nu|\eta, \ell)$.

¹⁴The symbol " $\stackrel{d}{=}$ " denotes a definition.

With probability one, the support of $H(\cdot|\eta, \ell)$ has a finite supremum $\bar{\nu}_{\eta\ell}$.¹⁵ We assume that H is not too concave, and its concavity is nondecreasing in ν :

No Cream Skimming Let H' and H'' denote the first and second derivatives of $H(\nu|\eta, \ell)$ with respect to ν . For all feasible pairs (η, ℓ) and for all ν in the interior of the support of $H(\cdot|\eta, \ell)$, (a) these derivatives exist and (b) $H''\nu/H'$ is greater than -1 and is weakly increasing in ν .

This property will imply that if bank a (for instance) lends to some agents with credit score η in location ℓ in region B , then bank a prefers to charge an interest rate that is low enough to deter bank b from lending to any agents in this group. Hence, in equilibrium bank b does not “cream skim”: lend to agents with high private types ν but not to all agents. This fact allows us to solve analytically for the interest rates that the banks charge for every credit score, location, and region. It is consistent with the observation of Agarwal and Hauswald [1] that internet lenders charge low rates partly in order to prevent cream skimming:

Arm’s-length debt is less readily available but carries lower rates because competition among symmetrically informed banks, which rely on public information, not only drive down its price but also restrict access to credit to minimize adverse selection. (Agarwal and Hauswald [1, p. 2])

The following result shows that No Cream Skimming is equivalent to a particular assumption on the primitives of the model.

Lemma 2.3 *Let F' and F'' denote the first and second derivatives of $F(s_{priv}|s_{pub}, \ell)$ with respect to s_{priv} . Let ν' and ν'' denote the first and second derivatives of $\nu = \nu(s_{priv}|s_{pub}, \ell)$ with respect to s_{priv} . Assume these derivatives exist. Then No Cream Skimming holds if and only if, for all s_{priv} , s_{pub} , and ℓ , $\frac{F''}{F'\nu'} - \frac{\nu''\nu}{[\nu']^2}$ is greater than -1 and is weakly increasing in s_{priv} .*

The following property states that for any given public signal, one can find private signals that are strong enough that make an agent at least as appealing as any other agent. For

¹⁵Since $\theta \leq 1$, $\bar{\nu}_{\eta\ell}$ is no greater than $1/\eta$. Since $\theta > 0$, $\eta = E(\theta|s_{pub}, \ell)$ is strictly positive for any s_{pub} that occurs with positive probability. Hence, $1/\eta$ is finite with probability one, so $\bar{\nu}_{\eta\ell}$ is as well.

instance, if an agent with several loan delinquencies (the public signal) has just inherited a large enough sum of money (the private signal), a bank can ignore her weak credit history.

Limit Irrelevance For any public signal s_{pub} , location ℓ , and $\varepsilon > 0$, there exists a private signal s_{priv} for which $E[\theta|s_{\text{pub}}, s_{\text{priv}}, \ell] > 1 - \varepsilon$.

This will imply that a remote bank lends to applicants whose credit scores exceed a location-dependent threshold.¹⁶ Indeed, Agarwal and Hauswald [1] find that the chance that a bank will approve an online loan is increasing in both the applicant's public credit quality and the bank's internal assessment, but the latter's effect is very small. Limit Irrelevance permits the depiction of our results using simple two-dimensional diagrams. We also consider what happens in the absence of this assumption.

We now produce an example that satisfies all of the above assumptions. Suppose that s_{priv} , s_{pub} , and ℓ are independent and each is uniformly distributed on the unit interval.¹⁷ This implies that $F(s_{\text{priv}}|s_{\text{pub}}, \ell) = s_{\text{priv}}$, so $F'' = 0$. Let the conditional distribution of θ given the two signals and location be $F(\theta|s_{\text{priv}}, s_{\text{pub}}, \ell) = \theta^{\frac{m}{1-m}}$ where $m = 1 - (1 - s_{\text{priv}})(1 - s_{\text{pub}})$. The mean of this distribution, $E(\theta|s_{\text{priv}}, s_{\text{pub}}, \ell)$, equals m . Hence,

$$\nu(s_{\text{priv}}|s_{\text{pub}}, \ell) = \frac{E(\theta|s_{\text{priv}}, s_{\text{pub}}, \ell)}{E(\theta|s_{\text{pub}}, \ell)} = \frac{1 - (1 - s_{\text{priv}})(1 - s_{\text{pub}})}{1 - \frac{1 - s_{\text{pub}}}{2}},$$

so $\nu'' = 0$ as well. No Cream Skimming then follows from Lemma 2.3. Limit Irrelevance holds since $\lim_{s_{\text{priv}} \rightarrow 1} E(\theta|s_{\text{priv}}, s_{\text{pub}}, \ell) = 1$. Since $\theta^{\frac{m}{m-1}}$ is strictly increasing in m , which is strictly increasing in s_{priv} , Private Signal Monotonicity holds. Public Signal Monotonicity holds since $F(\theta|s_{\text{pub}}, \ell) = \int_{s_{\text{priv}}=0}^1 F(\theta|s_{\text{priv}}, s_{\text{pub}}, \ell) ds_{\text{priv}}$.

2.2.1 Timing

We now describe each period in greater detail.

¹⁶Without Limit Irrelevance, a bank may offer loans in a given remote location to applicants with credit score η' but not to those whose credit scores are $\eta'' > \eta'$.

¹⁷This refers to the closed unit interval in the case of ℓ and the open interval in the case of s_{priv} and s_{pub} .

2.2.1.1 Period 1: Lending Stage

In period 1, the banks offer loans first to remote agents and then to local agents. That is, banks a and b first make simultaneous and public loan offers to agents who live in regions B and A , respectively. These offers can depend on an agent's credit score η and location ℓ , which are all the banks know. The banks then make simultaneous and public counter-offers to agents who live in regions A and B , respectively. These offers can depend not only on η and ℓ , but also on an applicant's private type ν and her offer (if any) from her remote bank. Each agent then chooses which, if any, offer to accept. As the banks are perfect substitutes from an agent's point of view, an agent will choose the bank that offers her the lowest gross interest rate as long as it does not exceed the project return ρ .

Let $x_{\eta\ell}^B$ equal one if bank a chooses to compete for agents with credit score η in location ℓ in region R and zero otherwise. Let $r_{\eta\ell}^B$ be the gross interest rate that bank a offers if $x_{\eta\ell}^B = 1$. We assume this rate does not exceed the gross project return ρ , since offering a rate above ρ is equivalent to not making an offer. If the agent did not receive an offer from bank a , then she is willing to pay bank b her gross project return ρ . Thus, with the convention that $r_{\eta\ell}^B$ equals ρ whenever bank a does not compete, $r_{\eta\ell}^B$ equals the willingness to pay of *any* agent. We assume there is an infinitesimal chance that the secondary loan market will be disrupted, forcing the bank to hold all of its loans to maturity. Since only bank b observes an agent's private type ν , this implies that a threshold strategy is optimal: bank b will bid $r_{\eta\ell}^B$ (and win) as long as an agent's private type ν exceeds a threshold $\underline{\nu}_{\eta\ell}^B$ of bank b 's choosing. Otherwise, bank b will not bid.

The banks swap roles with respect to agents who live in region A . Let $x_{\eta\ell}^A$ equal one if bank b chooses to compete for agents in region A with credit score η and location ℓ , and zero otherwise. Let $r_{\eta\ell}^A \leq \rho$ equal bank b 's bid in period 1 if $x_{\eta\ell}^A = 1$; set $r_{\eta\ell}^A = \rho$ otherwise. In period 2, bank a responds by choosing thresholds $\underline{\nu}_{\eta\ell}^A$ such that it will lend an agent in region A at interest rate $r_{\eta\ell}^A$ if and only if the agent's private type ν exceeds $\underline{\nu}_{\eta\ell}^A$.

Let C_a^B and X_a^B be the capital cost and realized value, respectively, of bank a 's loans to

region B :

$$C_a^B = \int_{\ell=0}^1 \int_{\eta=0}^1 x_{\eta\ell}^B H(\underline{\nu}_{\eta\ell}^B | \eta, \ell) dG(\eta, \ell)$$

$$X_a^B = \int_{\ell=0}^1 \int_{\eta=0}^1 x_{\eta\ell}^B r_{\eta\ell}^B \left[\eta L_{\ell}^B \int_{\nu=0}^{\underline{\nu}_{\eta\ell}^B} \nu dH(\nu | \eta, \ell) \right] dG(\eta, \ell)$$

Thus, C_a^B is the integral, over all credit scores η and locations ℓ in region B in which the bank competes (i.e., for which $x_{\eta\ell}^B = 1$), of the measure $H(\underline{\nu}_{\eta\ell}^B | \eta, \ell)$ of borrowers to whom bank a lends. Likewise, X_a^B is the integral, over all credit scores η and locations ℓ in region B in which bank a competes, of the interest rate $r_{\eta\ell}^B$ charged to these borrowers times their mean probability of repayment (the expression in square brackets).

Likewise, let C_a^A and X_a^A be the capital cost and realized value, respectively, of bank a 's loans to region A :

$$C_a^A = \int_{\ell=0}^1 \int_{\eta=0}^1 (1 - H(\underline{\nu}_{\eta\ell}^A | \eta, \ell)) dG(\eta, \ell)$$

$$X_a^A = \int_{\ell=0}^1 \int_{\eta=0}^1 r_{\eta\ell}^A \left[\eta L_{\ell}^A \int_{\nu=\underline{\nu}_{\eta\ell}^A}^{\bar{\nu}_{\eta\ell}} \nu dH(\nu | \eta, \ell) \right] dG(\eta, \ell)$$

The difference between C_a^A and C_a^B reflects the fact that bank a lends to borrowers in region A whose private types exceed bank a 's minimum threshold $\underline{\nu}_{\eta\ell}^A$, while it lends to borrowers in region B if and only if (1) it chooses to compete for them (i.e., only if $x_{\eta\ell}^B = 1$) and (2) their private types are below bank b 's minimum threshold $\underline{\nu}_{\eta\ell}^B$. This also explains the difference between X_a^A and X_a^B .

2.2.1.2 Period 2: Security Design Stage

In period 2, each bank designs one security. The number of shares of each security is normalized to one. We describe this process from the point of view of bank a ; bank b 's problem is analogous. First, bank a decides what portion of the loans of each identifiable group of borrowers to *securitize*: to include in the pool of assets that underlie its security. Bank a does not know the private types of its borrowers in region B . Hence, for any given credit score η and location ℓ , it must securitize the same proportion of loans to each type $\nu \in [0, \underline{\nu}_{\eta\ell}^B]$ of borrower in region B . Let this proportion be $p_{\eta\ell}^B$.

As for region A , since a borrower's private type ν is observed by bank a but not by the market, bank a will securitize a loan if and only if the borrower's private type ν is less than some threshold $\bar{\nu}_{\eta\ell}^A$, which must be at least as high as the minimum private type $\underline{\nu}_{\eta\ell}^A$ of borrowers in region A to whom the bank lends. The realized value of bank a 's securitized loans is $Y_a = Y_a^A + Y_a^B$ where

$$Y_a^A = \int_{\ell=0}^1 \int_{\eta=0}^1 r_{\eta\ell}^A \left[\eta L_{\eta\ell}^A \int_{\nu=\underline{\nu}_{\eta\ell}^A}^{\bar{\nu}_{\eta\ell}^A} \nu dH(\nu|\eta, \ell) \right] dG(\eta, \ell) \quad (2.2)$$

is the realized value of the bank's securitized local loans and

$$Y_a^B = \int_{\ell=0}^1 \int_{\eta=0}^1 p_{\eta\ell}^B x_{\eta\ell}^B r_{\eta\ell}^B \left[\eta L_{\eta\ell}^B \int_{\nu=0}^{\underline{\nu}_{\eta\ell}^B} \nu dH(\nu|\eta, \ell) \right] dG(\eta, \ell) \quad (2.3)$$

is the realized value of the bank's securitized remote loans. One obtains Y_a^A from X_a^A by replacing the supremum $\bar{\nu}_{\eta\ell}$ of private types ν in X_a^A with the upper bound $\bar{\nu}_{\eta\ell}^A$ on private types ν who are securitized. Similarly, one obtains Y_a^B from X_a^B by multiplying the integrand of the outer double integral in X_a^B by the proportion $p_{\eta\ell}^B$ of loans that are securitized.

After choosing which loans to securitize, each bank $i = a, b$ chooses a function φ_i which determines the ultimate payment per share made by the bank to a holder of its security as a function of the realized loan repayments Y_i of bank i 's securitized borrowers. We call $\varphi_i(Y_i)$ the *payout* of the security. As in DeMarzo and Duffie [16], we assume that φ_i is a nondecreasing function and that both the bank and the market have limited liability: $\varphi_i(y) \in [0, y]$ for all $y \geq 0$.

There is symmetric information at the security design stage. Why? Let $R(i)$ denote the region in which bank $i \in \{a, b\}$ is located. While the thresholds $\underline{\nu}_{\eta\ell}^{R(i)}$ and $\bar{\nu}_{\eta\ell}^{R(i)}$ are the private information of bank $i = a, b$, the market can infer the values Y_i^A and Y_i^B of bank i 's securitized local and remote loans that result from each pair of factor vectors (ζ^A, ζ^B) in the following way. First, we assume the market observes the measure $1 - H(\underline{\nu}_{\eta\ell}^{R(i)}|\eta, \ell)$ of bank i 's local borrowers for each credit score η and location ℓ , as well as the proportion $\frac{H(\bar{\nu}_{\eta\ell}^{R(i)}|\eta, \ell) - H(\underline{\nu}_{\eta\ell}^{R(i)}|\eta, \ell)}{1 - H(\underline{\nu}_{\eta\ell}^{R(i)}|\eta, \ell)}$ of these borrowers whom bank i securitizes. From these quantities, the market can infer the values $H(\underline{\nu}_{\eta\ell}^{R(i)}|\eta, \ell)$ and $H(\bar{\nu}_{\eta\ell}^{R(i)}|\eta, \ell)$ of the distribution function H at the two thresholds. We also assume that for each region R , the market observes the interest rates $r_{\eta\ell}^R$, the lending choices

$x_{\eta\ell}^R$, and the remote securitization proportions $p_{\eta\ell}^R$. The market can then use equations (2.2) and (2.3), or the corresponding equations for bank b , to compute Y_i^A and Y_i^B for any factor vectors ζ^A and ζ^B .

2.2.1.3 Period 3: Signalling Stage

In period 3, the banks and investors first see a common public signal $\sigma \in \mathfrak{R}^M$, with unconditional distribution function Ω . Each bank i then sees a private signal $u^i \in \mathfrak{R}_+^N$ of its local factor vector $\zeta^{R(i)} \in (0, 1)^K$. The local factor vector $\zeta^{R(i)}$ and the local signal u^i are drawn from a joint density $\gamma(\zeta^{R(i)}, u^i | \sigma)$, which can depend on the public signal σ as indicated by the notation. However, *conditional* on the public information σ , (ζ^A, u^a) and (ζ^B, u^b) are independent: the realization of (ζ^A, u^a) adds no information about the distribution of (ζ^B, u^b) and vice-versa. This is a flexible yet tractable way to permit common or correlated shocks to the two regions.

Let the distribution function of the private signal u^i conditional on the public signal σ be $\Psi(u^i | \sigma)$. We assume that for all public signals σ , private signals u^i close to the zero vector are observed with strictly positive probability:

$$\inf \{u^i \in \mathfrak{R}_+^N : \Psi(u^i | \sigma) > 0\} = 0.$$

Let $\Gamma(\zeta^{R(i)} | u^i, \sigma)$ be the conditional distribution of the factor vector $\zeta^{R(i)}$ given the private signal u^i and the public signal σ . A higher private signal u^i raises this distribution in the sense of first order stochastic dominance: if $u' \geq u''$, then for all ζ , $\Gamma(\zeta | u', \sigma) \leq \Gamma(\zeta | u'', \sigma)$. This implies that for any public signal σ , the worst news bank i can get about its security payout $\varphi_i(Y_i)$ occurs when its private signal u^i is zero. Finally, we assume that the conditional distribution $\Gamma(\zeta^{R(i)} | u^i, \sigma)$ is mutually absolutely continuous with respect to the signals (u^i, σ) .¹⁸

The assumption that the density γ and distributions Ψ and Γ are region-independent is for notational convenience. They could be replaced by γ^R , Ψ^R , and Γ^R with no change in the results, except for the proliferation of regional superscripts throughout the chapter.

¹⁸This means that the set of realizations of the factor vector $\zeta^{R(i)}$ that can occur with positive probability is independent of the signals u^i and σ .

After seeing their signals, the banks choose quantities of their securities to sell. Bank i 's quantity is denoted $q_i \in [0, 1]$. The market (which also sees the public signal σ) uses Bayes's rule to assign a price $p_i = E[\varphi_i(Y_i) | q_a, q_b, \sigma]$ to the security of bank $i = a, b$. This is a nonstandard signalling game since the market rationally uses information about bank i 's quantity q_i to infer information about bank i 's signal u^i , which may be relevant to the value of bank j 's security (as it may include some loans to borrowers in bank i 's region).

2.2.1.4 Period 4: Settlement Stage

In period 4, each borrower repays her loan if and only if her project succeeds. These repayments determine the value Y_i of bank i 's loan portfolio. Bank i then pays the liquidating dividend $\varphi_i(Y_i)$ to its investors. While periods 1 through 3 occur at the same point of real time, there is a unit of delay between periods 3 and 4.

2.2.2 Payoffs

A borrower who pays interest rate r gets $\rho - r$ if her project succeeds and zero otherwise. The banks are liquidity constrained: the discount factor of security buyers, which we normalize to one, exceeds the discount factor of the banks, which is denoted $\delta \in (0, 1)$.¹⁹ The two banks have the same cost of capital, which is normalized to one. In particular, suppose a bank lends c_1 units of capital in period 1 to borrowers who later repay the bank c_4 in period 4. Assume, moreover, that investors pay the bank c_3 in period 3 in return for a security that obligates the bank to pay the investors c'_4 in period 4. Then the payoff of investors in the bank's security is $c'_4 - c_3$, while the bank's payoff equals $c_3 - c_1 + \delta(c_4 - c'_4)$: its securitization proceeds c_3 , less its capital cost c_1 , plus its discounted loan repayments δc_4 , less its discounted payment $\delta c'_4$ to holders of its security. We assume the investors have at least 2ρ in capital to invest.²⁰

¹⁹This assumption, common in the prior literature, is thought to capture the typical reason cited for why banks sell loans: the availability of attractive alternative investments together with the existence of regulatory capital ratios (e.g., Gorton and Haubrich [27§III.B]).

²⁰Since each region has a unit measure of loan applicants, each with a project that returns ρ if it succeeds, the securities of the two banks cannot be worth more than 2ρ to the market.

2.2.3 Summary

We now briefly summarize the key features of the model. We focus on region B ; analogous choices are made simultaneously in region A with the banks' roles swapped. Consider the group of agents with a given credit score η and location ℓ . In period 1, bank a either offers each such agent a loan at the common interest rate $r_{\eta\ell}^B \in [0, \rho]$ or refrains from competing (whence we set $r_{\eta\ell}^B = \rho$). Bank b then lends, at the interest rate $r_{\eta\ell}^B$, to those agents in the group whose private types exceed a threshold $\underline{\nu}_{\eta\ell}^B$ of bank b 's choosing. Agents with lower private types accept a 's offer, if any.

In period 2, bank a chooses a proportion $p_{\eta\ell}^B \in [0, 1]$ of its loans to the group to securitize. Bank b securitizes its loans to group members whose private types fall below a threshold $\bar{\nu}_{\eta\ell}^B$ of bank b 's choosing. Each bank i also specifies a payout function φ_i .

In period 3, each bank $i = a, b$ sees a signal u^i of its local factor vector and then chooses a quantity $q_i \in [0, 1]$ of shares to sell. The market rationally assigns a price p_i to bank i 's security using Bayes's rule. In period 4, project returns are realized and successful borrowers repay their loans. Each bank i then pays $\varphi_i(Y_i)$ per share to its security holders, where Y_i equals the repayments of bank i 's securitized loans.

2.3 Base Model: No Securitization

We first analyze a base model without securitization: banks must hold all of their loans to maturity. Bank a 's payoff in the base model is simply its discounted loan repayments less its cost of lent capital: $\delta E(X_a^A + X_a^B) - C_a^A - C_a^B$. Bank b 's payoff is analogous. In particular, if a bank lends, at a gross interest rate r , to a borrower with credit score η and private type ν living in location ℓ in region $R \in \{A, B\}$, its expected profit is $\delta r \eta \nu E(L_\ell^R) - 1$: the discounted interest payment δr times the probability $\eta \nu E(L_\ell^R)$ of project success, less the unitary cost of capital.

In the base model, banks lend only to local agents and extract the full surplus. This is due to the winner's curse: the banks have the same expected payoff from lending to a given agent, but the agent's local bank has superior information about this payoff. Since, by assumption,

the local bank makes the second offer, it will slightly underbid the remote bank on profitable loans but refrain from bidding on unprofitable ones. Knowing this, a bank will not make any offers to agents who are not in its region.

Lemma 2.4 *Without the option of securitization, each bank lends only to agents who reside in its own region. Moreover, each borrower's payoff is zero: the gross interest rate on every loan equals the gross project return ρ . An agent gets a loan if and only if her discounted expected gross project return, $\delta\rho\nu E(L_\ell^R)$, exceeds the bank's unitary cost of capital.*

Without securitization, an agent gets a loan if and only her expected project return exceeds a common threshold. Hence, the allocation of capital to projects is efficient: one agent receives a loan while another does not if and only if the first has a higher expected project return than the second. This efficiency property will not hold with securitization, since a bank may prefer not to lend to a creditworthy agent whom it knows well. Intuitively, the bank's private information about this borrower's repayment probability worsens the lemons problem the bank faces in selling its security.

Our conclusion that all lending is local and the loan allocation is efficient relies on our assumption that the remote bank makes the first offer, followed by the local bank. However, Sharpe [58] obtains the same result with the reverse timing. He assumes that the remote bank sees not the local bank's offer but rather its offer function: the function from the local bank's signal to its interest rate. If, in addition, the remote bank has no private information about the applicant, then the local bank always posts an offer function that is low enough to make it unprofitable for the remote bank to compete because of a winner's curse (Sharpe [58, Proposition 2, p. 1078]).

2.4 Full Model

We now turn to the full model, with securitization. We first show that the signalling subgame has a unique separating equilibrium. We then derive formulas for a bank's benefit of securitizing a given loan and of lending to a given borrower when securitization is an option.

Finally, we show that any equilibrium of the full model must have a certain intuitive form. We then turn to several computed examples.

2.4.0.1 The Signalling Subgame

Let $\phi_i(u^i, u^j, \sigma) = E[\varphi_i(Y_i) | u^i, u^j, \sigma]$ be the expected payout of the security of bank $i \in \{a, b\}$, conditional on the signals. (“ j ” refers to the other bank.) Let $p_i(q_i, q_j, \sigma)$ be the price offered by the market per unit of bank i 's security as a function of the quantities of shares sold by the two banks and the public signal. Bank i 's expected securitization profits $\pi^i(u^i, q_i, \sigma)$, conditional on its signal u^i and quantity q_i and the public signal σ equal the expectation (over all opposing signal vectors u^j) of bank i 's gross revenue $q_i p_i(q_i, q_j(u^j), \sigma)$ from selling q_i units of the security less its discounted expected payment to the buyers, $\delta q_i \phi_i(u^i, u^j, \sigma)$:

$$\pi^i(u^i, q_i, \sigma) = \int_{u^j \in \mathbb{R}_+^N} (q_i [p_i(q_i, q_j(u^j), \sigma) - \delta \phi_i(u^i, u^j, \sigma)]) d\Psi(u^j).$$

Definition 2.5 *A Bayes-Nash equilibrium of this game is a pair (q_a, q_b) of measurable quantity functions and a pair (p_a, p_b) of measurable price functions such that:*

1. for $i = a, b$, $q_i(u^i, \sigma) \in \arg \max_q \pi^i(u^i, q, \sigma)$ almost surely;
2. for $i = a, b$, $p_i(q_i(u^i), q_j(u^j), \sigma) = E[\phi_i(u^i, u^j, \sigma) | q_i(u^i), q_j(u^j), \sigma]$ almost surely;

The equilibrium is separating if, in addition,

3. for $i = a, b$, $p_i(q_i(u^i), q_j(u^j), \sigma) = \phi_i(u^i, u^j, \sigma)$ almost surely.

We restrict to separating equilibria, which satisfy conditions 1 and 3 above. This restriction uniquely determines the banks' behavior and profits. Let $\pi^i(u^i, \sigma) = \pi^i(u^i, q_i(u^i, \sigma), \sigma)$ and $\widehat{\phi}_i(u^i, \sigma) = \int_{u^j \in \mathbb{R}_+^N} \phi_i(u^i, u^j, \sigma) d\Psi(u^j | \sigma)$ be bank i 's securitization profits and expected security payout, both conditioned only on bank i 's signal u^i and the public signal σ . (In general, $\pi^i(u^i, \sigma)$ may depend on the equilibrium.) The following characterization extends the result of DeMarzo and Duffie [16, eq. (4), p. 79, and Prop. 10, p. 88], which assumes a single bank, to the case of two banks.²¹

²¹It is easy to see that this result generalizes to any finite number of banks.

Lemma 2.6 *The above double signalling game has a unique separating equilibrium. In it, bank i 's expected securitization profits conditional on its signal u^i and the public signal σ are $\pi^i(u^i, \sigma) = (1 - \delta) \widehat{\phi}_i(0, \sigma)^{\frac{1}{1-\delta}} \widehat{\phi}_i(u^i, \sigma)^{-\frac{\delta}{1-\delta}}$. Moreover, each bank i 's optimal security design is debt: $\varphi_i(Y_i) = \min\{m_i, Y_i\}$ for some $m_i \in \mathfrak{R}_+$.*

2.4.0.2 The Benefits of Securitization

Consider either bank $i \in \{a, b\}$. By Lemma 2.6, the realized payout of the bank's security is $\min\{m_i, Y_i\}$ where $Y_i = Y_i^A + Y_i^B$ is the realized value of bank i 's securitized loans and m_i is the face value (promised repayment) of the security. Consequently, the expected payout $\widehat{\phi}_i(u^i, \sigma)$ of bank i 's security given its signal u^i and the public signal σ is $E[\min\{m_i, Y_i\} | u^i, \sigma]$. Bank i 's expected payoff Π_i is the discounted expected return of its loans, less its cost of lending, plus its net securitization profits. By Lemma 2.6,

$$\Pi_i = \delta E(X_i^A + X_i^B) - C_i^A - C_i^B + (1 - \delta) E\left(\frac{\widehat{\phi}_i(0, \sigma)^{\frac{1}{1-\delta}}}{\widehat{\phi}_i(u^i, \sigma)^{\frac{\delta}{1-\delta}}}\right)$$

In order to understand bank i 's incentives to lend to a given agent, one must first consider its benefit from securitizing the agent's loan. To study this, we hold fixed the bank's loan portfolio, and consider the effect of adding a single infinitesimal loan to the bank's security.

Suppose the recipient of this loan has credit score η and private type ν , and lives in location ℓ in region $R \in \{A, B\}$.²² Let r be the gross interest rate that she must pay if her project succeeds, which occurs with probability $\eta\nu L_\ell^R$. By Lemma 2.6, and the law of iterated expectations, bank i 's expected securitization profits are $(1 - \delta) E\left[E\left(\widehat{\phi}_i(0, \sigma)^{\frac{1}{1-\delta}} \widehat{\phi}_i(u^i, \sigma)^{-\frac{\delta}{1-\delta}} \middle| \sigma\right)\right]$, where the outer expectation is taken with respect to the public signal σ and the inner conditional expectation is taken with respect to the private signal u^i . The effect, on the bank's profits Π_i , of adding the borrower to the bank's security is thus

$$\Delta\Pi_i = (1 - \delta) E\left[E\left(\frac{\widehat{\phi}_i(0, \sigma)^{\frac{1}{1-\delta}}}{\widehat{\phi}_i(u^i, \sigma)^{\frac{\delta}{1-\delta}}} \left(\frac{\Delta\widehat{\phi}_i(0, \sigma)}{\widehat{\phi}_i(0, \sigma)} - \delta \frac{\Delta\widehat{\phi}_i(u^i, \sigma)}{\widehat{\phi}_i(u^i, \sigma)}\right) \middle| \sigma\right)\right]. \quad (2.4)$$

where for any quantity Q , ΔQ denotes the change in Q that results from adding the loan.

²²We assume that the market knows the private type ν since it can infer the set of private types that each bank securitizes (p. 37).

The terms $\Delta\widehat{\phi}_i(0, \sigma)$ and $\Delta\widehat{\phi}_i(u^i, \sigma)$ measure the loan's effect on the expected gross return $\widehat{\phi}_i(u^i, \sigma) = E[\min\{m_i, Y_i\}|u^i, \sigma]$ of the security in two cases: when the bank's private signal is zero, and when it takes a generic value u^i . In particular, by Lemma 2.6, adding the loan is beneficial insofar as it raises the gross return of the security in the worst case, or lowers it in the generic case. Since higher signals u^i entail higher values of Y_i in a first order stochastic dominance sense, $\widehat{\phi}_i(0, \sigma)$ cannot exceed $\widehat{\phi}_i(u^i, \sigma)$. Thus, roughly speaking, loans that shrink (raise) the gap between $\widehat{\phi}_i(u^i, \sigma)$ and $\widehat{\phi}_i(0, \sigma)$ must raise (lower) the bank's securitization profits.

The term $\Delta\widehat{\phi}_i(u^i, \sigma) = \Delta E[\min\{m_i, Y_i\}|u^i, \sigma]$ measures the effect of the loan on the bank's expected payment to its security holders, conditional on the signals u^i and σ . This effect occurs entirely through the loan's impact on the realized value Y_i of the bank's securitized loans. First, the security defaults when its face value m_i exceeds the value Y_i of the underlying loans. In this event, the loan raises the security payout by ΔY_i . Second, the loan lowers the chance of default by raising the realized value of the loan portfolio Y_i when this value lies slightly below the face value of the security, m_i . This effect is approximately equal to the product of two terms: the loss $m_i - Y_i$ from default and the probability that Y_i is slightly below m_i . Since both terms are close to zero, this second effect is zero to first order. Hence, the only effect is the first:

$$\Delta\widehat{\phi}_i(u^i, \sigma) = E[1(m_i > Y_i) \Delta Y_i | u^i, \sigma], \quad (2.5)$$

where $1(m_i > Y_i)$ equals one if $m_i > Y_i$ (if the security defaults) and zero otherwise.

Finally, by (2.1), the increase in the value Y_i of the underlying assets from adding the borrower is a weighted sum of the macroeconomic factors ζ_k^R that affect region R :

$$\Delta Y_i = r\eta\nu L_\ell^R = r\eta\nu \sum_{k=1}^K \alpha_{k\ell}^R \zeta_k^R. \quad (2.6)$$

Substituting (2.5) and (2.6) into (2.4) and using Lemma 2.6, we find that the effect of securitizing the additional borrower on the bank's payoff is

$$\Delta\Pi_i = r\eta\nu\Omega_{i\ell}^R, \quad (2.7)$$

where

$$\Omega_{i\ell}^R = E(\Omega_{i\ell}^R(\sigma)), \quad (2.8)$$

$$\Omega_{i\ell}^R(\sigma) = E \left[\pi^i(u^i, \sigma) \mid \sigma \right] \sum_{k=1}^K \alpha_{k\ell}^R \left[\Lambda_{ik}^{R0}(\sigma) - \delta \Lambda_{ik}^R(\sigma) \right],$$

$$\Lambda_{ik}^{R0}(\sigma) = E \left(\frac{E \left[1(m_i > Y_i) \zeta_k^R \mid u^i = 0; \sigma \right]}{\widehat{\phi}_i(0, \sigma)} \mid \sigma \right),$$

and

$$\Lambda_{ik}^R(\sigma) = E \left(\frac{\pi^i(u^i, \sigma)}{E(\pi^i(u^i, \sigma) \mid \sigma)} \frac{E \left[1(m_i > Y_i) \zeta_k^R \mid u^i, \sigma \right]}{\widehat{\phi}_i(u^i, \sigma)} \mid \sigma \right).$$

By (2.7), profits from securitizing the loan are the product of four terms. The first is the gross interest rate r : ceteris paribus, it is more profitable to securitize loans that have a higher face value. The second is η : it is more profitable to securitize the loans of borrowers with higher credit scores. The third is ν : borrowers with high private types are also more profitable. The final term is $\Omega_{i\ell}^R$ which, by construction, must equal the change in securitization profits from adding a loan for which the product $r\eta\nu$ of the first three terms equals one.

By (2.8), $\Omega_{i\ell}^R$ is the expectation, over all public signals σ , of the change $\Omega_{i\ell}^R(\sigma)$ in securitization profits from adding a loan for which the product $r\eta\nu = 1$ and the public signal is σ . $\Omega_{i\ell}^R(\sigma)$, in turn, is the product of the bank's conditional (on the public signal σ) expected securitization profits $E[\pi^i(u^i, \sigma) \mid \sigma]$ and the sum, over all factors k , of the borrower's factor loading $\alpha_{k\ell}^R$ times the scaled difference between two terms: $\Lambda_{ik}^{R0}(\sigma)$ and $\delta \Lambda_{ik}^R(\sigma)$.

The term $\Lambda_{ik}^{R0}(\sigma)$ is the proportional increase in the lowest conditional expected security payout, $\widehat{\phi}_i(0, \sigma)$, that results from increasing the value Y_i of the security's underlying assets by one dollar with probability ζ_k^R .²³ Thus, $\sum_{k=1}^K \alpha_{k\ell}^R \Lambda_{ik}^{R0}(\sigma)$ captures the additional loan's proportional effect on this *worst-case* security payout that is due to the loadings $\alpha_{k\ell}^R$ of the borrower's repayment probability on various macroeconomic factors ζ_k^R . Likewise, $\Lambda_{ik}^R(\sigma)$ is a weighted average over signal vectors u^i of the proportional increase in the conditional expected security payout $\widehat{\phi}_i(u^i, \sigma)$ that results from increasing the value Y_i of the security's underlying assets by one dollar with probability ζ_k^R . Thus, $\sum_{k=1}^K \alpha_{k\ell}^R \Lambda_{ik}^R(\sigma)$ captures the proportional effect of the additional loan on this *weighted average* security payout that results from the loadings of the borrower's repayment probability on the various macroeconomic factors that affect region R .

²³In the numerator of $\Lambda_{ik}^{R0}(s)$, the default indicator variable $1(m_i > Y_i)$ is present because the additional borrower affects the security value only in the event of default.

By Lemma 2.6, for any public signal σ , the bank's securitization profits are increasing in the expected security payoff conditional on the worst signal vector $u^i = 0$ and decreasing in the expected security payoff for a generic signal vector u^i . For this reason, $\Lambda_{ik}^{R0}(\sigma)$ enters positively in $\Omega_{i\ell}^R(\sigma)$ while $\Lambda_{ik}^R(\sigma)$ enters negatively. The discount factor δ multiplying $\Lambda_{ik}^R(\sigma)$ captures the bank's preference for liquidity: the lower is δ , the stronger are the bank's liquidity needs, and thus the more likely it is that securitizing the additional loan will be worthwhile.

The above results allow us to derive a concise expression for the total expected gross return to bank $i \in \{a, b\}$ from lending to an agent with credit score η and private type ν who lives in location ℓ in region $R \in \{A, B\}$, when securitization is an option. This expected return has two parts. The first is the expected discounted loan repayment by the borrower, $\delta r \eta \nu E(L_\ell^R)$: the discounted interest rate δr times the probability $\eta \nu E(L_\ell^R)$ that the loan will be repayed. The second is the value of the bank's option to securitize the loan. By (2.7), bank i earns an additional $r \eta \nu \Omega_{i\ell}^R$ from securitizing the agent's loan, which it will do if and only if $\Omega_{i\ell}^R > 0$. For any real number c , let c^+ denote the positive part of c : $c^+ = \max\{0, c\}$. The value of the securitization option is $r \eta \nu (\Omega_{i\ell}^R)^+$, so the bank's gross return from lending to the borrower is

$$r \eta \nu \left[\delta E(L_\ell^R) + (\Omega_{i\ell}^R)^+ \right]. \quad (2.9)$$

Bank i knows the private type ν of the borrower only if she lives in the bank's home region. This is a disadvantage of remote lending. However, there is also a potential advantage: the bank does not have private information about remote shocks. Hence, it faces a lemons problem in reselling local loans but not remote loans. In addition, the bank has a preference for liquidity: $\delta < 1$. For the last two reasons, it is always profitable to securitize a remote loan:

Lemma 2.7 *Let $i \neq j$ be the two banks. In any equilibrium, it is profitable for bank i to securitize all of its remote loans: $\Omega_{i\ell}^{R(j)} > 0$.*

2.4.0.3 Main Results

We present results for region B . Identical results hold for region A upon replacing “ a ” with “ b ” and vice versa. Let $r_{\eta\ell}^{B*}$ denote the *detering rate*: the interest rate, offered by bank

a , that makes bank b just willing not to lend to the agent with the highest private type (for whom $\nu = \bar{\nu}_{\eta\ell}$) among those with credit score η living in location ℓ in region B . By equation (2.9), bank b 's gross expected return from lending to this borrower at the interest rate r is $r\eta\bar{\nu}_{\eta\ell} \left[\delta E(L_\ell^B) + (\Omega_{b\ell}^B)^+ \right]$. Setting this equal to the bank's unitary cost of capital and solving for r , we obtain the deterring rate:

$$r_{\eta\ell}^{B*} = (\eta\bar{\nu}_{\eta\ell})^{-1} \left[\delta E(L_\ell^B) + (\Omega_{b\ell}^B)^+ \right]^{-1} > 0. \quad (2.10)$$

Now consider the set of borrowers with credit score η in location ℓ in region B . No Cream Skimming implies that if bank a competes for these borrowers, it prefers to lend to all of them: to prevent bank b from skimming the best (highest- ν) borrowers in the group. This requires bank a to bid an interest rate that is no higher than the deterring rate $r_{\eta\ell}^{B*}$. In addition, bank a cannot charge more than the gross project return ρ , which is the most any borrower will pay. On the other hand, any interest rate below the lesser of ρ and $r_{\eta\ell}^{B*}$ permits bank a to capture all of the borrowers in this group. Hence, if bank a competes for these borrowers, it will offer the interest rate $r_{\eta\ell}^B = \min \{ \rho, r_{\eta\ell}^{B*} \}$. By equation (2.9), Lemma 2.7, and the fact that $E(\nu|\eta, \ell) = 1$, bank a 's profits from lending a unit of capital to this group are

$$\pi_{\eta\ell}^B = \min \{ \rho, r_{\eta\ell}^{B*} \} \left[\delta E(L_\ell^B) + \Omega_{a\ell}^B \right] \eta - 1. \quad (2.11)$$

Our first result, which does not assume Limit Irrelevance, is as follows.

Theorem 2.8 *Consider the group of agents with credit score η in location ℓ in region B .*

1. *Suppose $\pi_{\eta\ell}^B < 0$. In this case,*

(a) *bank a does not compete for this group;*

(b) *if bank b 's estimate $\nu\eta$ of an agent's type θ exceeds $\left(\rho \left[\delta E(L_\ell^B) + (\Omega_{b\ell}^B)^+ \right] \right)^{-1}$, bank b offers her a loan at an interest rate equal to the gross project return ρ , and the agent accepts.²⁴ Else bank b does not offer the agent a loan. Bank b securitizes all borrowers in this group to whom it lends if $\Omega_{b\ell}^B > 0$ and none of them if $\Omega_{b\ell}^B < 0$.*

²⁴By definition, $\nu\eta = E(\theta|s_{priv}, s_{pub}, \ell)$ (p. 32).

2. Suppose $\pi_{\eta\ell}^B > 0$. In this case,

- (a) bank a offers to lend to each agent in the group at the common interest rate $r_{\eta\ell}^B = \min \{ \rho, r_{\eta\ell}^{B*} \}$;
- (b) bank b makes no offers to this group;
- (c) all agents in the group accept bank a's offer; and
- (d) bank a securitizes all of them.

Consider the set of borrowers in a given location ℓ in region B . Theorem 2.8 characterizes the outcome, in the loan market, of borrowers with a *given* credit score η in this set. It does not show how this outcome varies by the credit score η . We now turn to this important question.

The key difficulty is that bank a's profit $\pi_{\eta\ell}^B$ from lending is not necessarily monotonic in the agent's credit score η . This profit is increasing in the deterring rate $r_{\eta\ell}^{B*}$ (equation (2.11)) which, in turn, is decreasing in the supremum $\bar{\nu}_{\eta\ell}$ of the agent's possible private types ν (equation (2.10)). However, we have not specified how the supremum $\bar{\nu}_{\eta\ell}$ varies with the credit score η . Limit Irrelevance pins this down in a particular way: $\bar{\nu}_{\eta\ell}$ equals the inverse of the credit score η .²⁵ By (2.10), the deterring rate, which we now call simply r_{ℓ}^{B*} , is independent of the credit score η :

$$r_{\ell}^{B*} = \left[\delta E(L_{\ell}^B) + (\Omega_{b\ell}^B)^+ \right]^{-1}. \quad (2.12)$$

We now present our second result.

Theorem 2.9 *Assume Limit Irrelevance. Define the threshold*

$$\eta_{\ell}^B = \frac{1}{\min \{ \rho, r_{\ell}^{B*} \} \left[\delta E(L_{\ell}^B) + \Omega_{a\ell}^B \right]}. \quad (2.13)$$

1. If $\eta < \eta_{\ell}^B$, then $\pi_{\eta\ell}^B < 0$: bank a does not compete for this group. If bank b's estimate $\nu\eta$ of an agent's type θ exceeds $\left(\rho \left[\delta E(L_{\ell}^B) + (\Omega_{b\ell}^B)^+ \right] \right)^{-1} = r_{\ell}^{B*} / \rho$, bank b offers her a loan at an interest rate equal to the gross project return ρ , and the agent accepts. Else bank b does not offer the agent a loan. Bank b securitizes all borrowers in this group to whom it lends if $\Omega_{b\ell}^B > 0$ and none of them if $\Omega_{b\ell}^B < 0$.

²⁵This is because $\bar{\nu}_{\eta\ell} = \eta^{-1} \sup_{s_{\text{priv}}} E[\theta | s_{\text{pub}}, s_{\text{priv}}, \ell] = 1$.

2. If $\eta > \eta_\ell^B$, then $\pi_{\eta\ell}^B > 0$: bank a offers all borrowers in this group the same interest rate $\min\{\rho, r_\ell^{B*}\}$. Bank b does not compete and all agents accept bank a 's offer. Moreover, bank a securitizes all loans to this group.

Proof: By Limit Irrelevance, $\bar{v}_{\eta\ell} = 1/\eta$, so $r_{\eta\ell}^{B*} = r_\ell^{B*}$. By equations (2.11) and (2.13), $\pi_{\eta\ell}^B = \eta/\eta_\ell^B - 1$. Hence, $\pi_{\eta\ell}^B \geq 0$ as $\eta \geq \eta_\ell^B$. The rest follows from Theorem 2.8 and equation (2.12). Q.E.D.

Under Limit Irrelevance, bank a lends to an agent in region B if and only if her credit score η exceeds the location-dependent credit threshold η_ℓ^B . This threshold is decreasing in bank a 's securitization profits, as captured by $\Omega_{a\ell}^B$, and weakly decreasing in bank b 's securitization profits, as captured by $(\Omega_{b\ell}^B)^+$. If bank a 's securitization profits are low relative to those of bank b , it is harder for bank a to compete with bank b . Bank a responds by competing for fewer borrowers in location ℓ : it raises its threshold.²⁶

By part 2 of Theorem 2.9 and equation (2.12)), if an agent's credit score is above a 's threshold, bank a offers the interest rate $\min\left\{\rho, \left[\delta E(L_\ell^B) + (\Omega_{b\ell}^B)^+\right]^{-1}\right\}$. This is weakly decreasing in bank b 's securitization profits, as captured by $(\Omega_{b\ell}^B)^+$.²⁷ Intuitively, if bank b is eager to securitize loans to the given location, then bank a must offer a low interest rate in order to keep bank b out.

A key prediction of Theorem 2.9 is that a bank will use a credit score threshold in deciding on remote loan applications. This feature survives a considerable weakening of Limit Irrelevance. As long as bank a 's profit $\pi_{\eta\ell}^B$ equals zero at a unique value of η , a threshold policy is optimal.²⁸ By equations (2.10) and (2.11), a sufficient condition for this is that $\bar{v}_{\eta\ell}$ - the maximum proportional increase in the agent's expected type θ that comes from learning her private signal σ_{priv} - be decreasing in η . This seems plausible; for instance, knowing that a loan applicant comes from a good family would seem to raise her chances of repaying a loan by a smaller proportion if her credit record is already quite strong.

²⁶This occurs, in particular, if location ℓ in region B has low loadings on factors about which bank b will be well informed when it decides how much of its security to sell.

²⁷By part 2 of Theorem 2.9 and equation (2.12)), bank a offers the interest rate

²⁸If it crosses zero, it must cross from below since $\pi_{0\ell}^B = -1$.

2.5 Illustrations

We now discuss the implications of Theorem 2.9 for the effects of securitization, comparative statics, and efficiency under Limit Irrelevance. We illustrate these results in a series of figures. The figures - but not the discussion - rely on the following additional assumptions.

A1 Bank b would lend to some agents in the absence of securitization: its discounted return $\delta\rho E(L_\ell^B)$ from lending to the best agent (for whom $\eta\nu = 1$) exceeds the bank's unitary cost of capital. By equation (2.12), this implies that the deterring rate r_ℓ^{B*} is less than the gross project return ρ , so bank a lends at the deterring rate. Hence, by equations (2.12) and (2.13), bank a 's credit score threshold under securitization is

$$\eta_\ell^B = \frac{\delta E(L_\ell^B) + (\Omega_{b\ell}^B)^+}{\delta E(L_\ell^B) + \Omega_{a\ell}^B}. \quad (2.14)$$

A2 Bank b benefits from securitization: $\Omega_{b\ell}^B > 0$.

A3 Bank a benefits more than bank b from securitization: $\Omega_{a\ell}^B > \Omega_{b\ell}^B$: Without this condition, $\eta_\ell^B \geq 1$, so bank a will not lend in the location.

In Figure 2.1, each agent in the location corresponds to a point in the unit square.²⁹ The agent's credit score η , which equals bank a 's estimate of her type θ , appears on the horizontal axis. Bank b 's estimate $\eta\nu$ of θ appears on the vertical axis.³⁰ While bank a sees only an agent's horizontal coordinate, bank b sees both.

In the absence of securitization, agents in areas A_0 and A_3 borrow from bank b at the interest rate ρ , while other agents do not get loans (Lemma 2.4). With securitization, agents in areas A_3 and A_4 borrow from bank a at the deterring rate $r_\ell^{B*} < \rho$, while those in areas A_0 and A_1 get loans from bank b at the interest rate ρ .

2.5.1 The Effects of Securitization

A comparison of Lemma 2.4 and Theorem 2.9 reveals the following effects of securitization, which are discussed in section 2.1.

²⁹The applicants are not uniformly distributed throughout the square.

³⁰While the figure permits η and $\eta\nu$ each to take any value in the unit interval, some of these values may have zero probability.

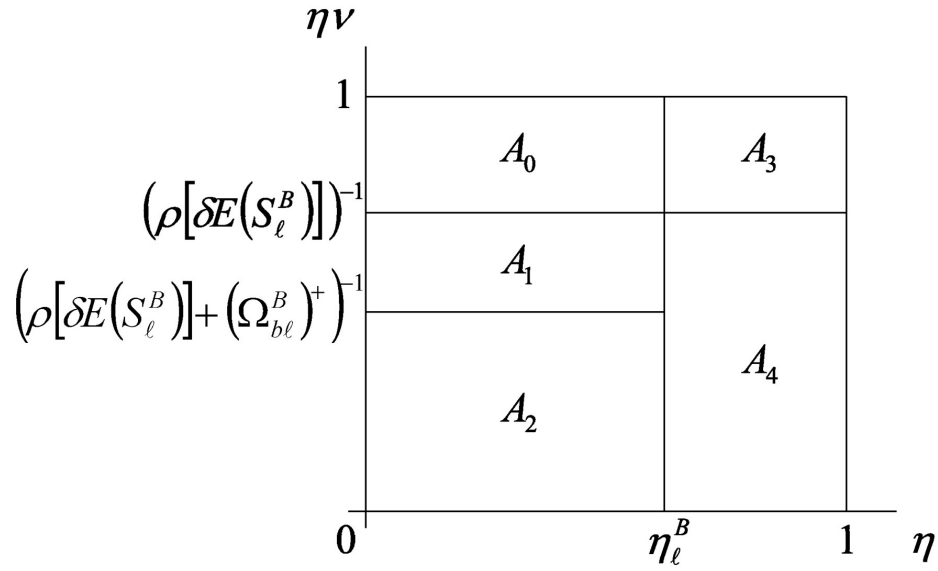


Figure 2.1 Effects of Securitization under Limit Irrelevance. A given location ℓ in region B is depicted. The credit rating η appears on the horizontal axis while bank b 's estimate $\eta\nu$ of an applicant's type θ is depicted on the vertical axis. The figure assumes that $\delta\rho E(S_\ell^B) > 1$ and $\Omega_{al}^B > \Omega_{bl}^B > 0$. Without securitization, applicants in regions A_0 and A_3 receive loans from bank b at the interest rate ρ . Those in regions A_1 , A_2 , and A_4 do not receive loans. With securitization, applicants in regions A_3 and A_4 receive loans from bank a at the interest rate $r_{\eta\ell}^{B*} < \rho$. Applicants in regions A_0 and A_1 receive loans from bank b at the interest rate ρ , while applicants in region A_2 do not receive loans.

1. **Securitization Stimulates Lending.** By connecting agents with liquid investors, securitization expands the set of borrowers.³¹ In Figure 2.1, areas A_1 and A_4 are added.
2. **Securitization Favors Remote Lending.** Remote bank a lends to location ℓ only if it can securitize its loans.
3. **Remote Borrowers have Strong Observables but High Conditional Default Rates.** In Figure 2.1, the applicants who get remote loans are those whose credit scores η exceed bank a 's threshold η_ℓ^B : they have strong observables. Now consider an otherwise identical neighborhood ℓ' in which the bank a 's securitization profits $\Omega_{a\ell'}^B$ are higher than in location ℓ . This raises bank a 's threshold: $\eta_{\ell'}^B > \eta_\ell^B$. The only applicants who are affected are those whose credit scores η lie between the two thresholds. In location ℓ , these applicants all get remote loans. In location ℓ' they get local loans, but only if bank b 's estimate $\eta\nu$ of their type is at least $\left(\rho [\delta E(L_\ell^B)] + (\Omega_{b\ell}^B)^+\right)^{-1} > 0$. Thus, ceteris paribus, a remote borrower with a given credit score η has an expected type $\eta\nu$ that is no higher, and sometimes strictly lower, than the expected type of a local borrower with the same credit score.
4. **Securitization Lets Borrowers with Strong Observables Get Cheap Remote Loans.** Securitization lowers the interest rate paid by agents with high credit scores (above η_ℓ^B) to $\min\{\rho, r_\ell^{B*}\}$ while leaving unchanged the interest rate ρ paid by agents with lower credit scores.
5. **Securitization Raises Conditional and Unconditional Default Rates.** Securitization expands lending to a set of borrowers (in Figure 2.1, those in areas A_1 and A_4) whose expected types $\eta\nu$ are uniformly lower than those of agents who borrow without securitization (those in areas A_0 and A_3). This raises both conditional (on η) and unconditional default rates.
6. **Securitized Loans Have Higher Conditional Default Rates than Retained Loans.** For any given credit score η , securitized loans have higher mean default rates

³¹All effects described in sections 2.5.1 through 2.5.4 are intended in the weak sense: the set of borrowers weakly increases, etc. In the figures, these effects are strict.

than retained loans. More precisely, let us compare two locations ℓ and ℓ' in region B . Assume bank b securitizes its loans to location ℓ' but not to location ℓ : $\Omega_{b\ell}^B < 0 < \Omega_{b\ell'}^B$. In all other respects, the two locations are identical. The comparison is depicted in Figure 2.2. For credit scores below η_{ℓ}^B , retained loans consist of area A_0 in location ℓ , while securitized loans consist of areas A_0 and A_1 in location ℓ . For each credit score, the securitized loans have a lower conditional expected type $\eta\nu$ than the retained loans. For credit scores above η_{ℓ}^B , all loans are securitized in both locations. Hence, for each credit score η for which there are retained loans in one location and securitized loans in the other, the latter group has a higher conditional default rate.

2.5.2 Higher Securitization Profits for the Local Bank

Suppose bank b 's securitization profits rise from $\Omega_{b\ell}^B$ to $\tilde{\Omega}_{b\ell}^B$. Since it is now harder to deter bank b from cream-skimming, bank a does so less often: it raises its credit score threshold from η_{ℓ}^B to $\tilde{\eta}_{\ell}^B = \frac{\delta E(L_{\ell}^B) + (\tilde{\Omega}_{b\ell}^B)^+}{\delta E(L_{\ell}^B) + \Omega_{a\ell}^B}$ (equation (2.14)). Theorem 2.9 implies the following effects, which are illustrated in Figure 2.3.

1. **Relatively More Local Lending.** Bank b lends more, while bank a lends less. In Figure 2.3, Bank b picks up borrowers in areas A_2 and A_5 . Bank a stops lending to areas A_4 through A_6 and is left with only A_7 and A_8 .
2. **More Lending to Diamonds in the Rough.** The set of borrowers grows to include those with credit scores below a 's threshold η_{ℓ}^B , whose expected types lie between b 's new and old thresholds. This is area A_2 in Figure 2.3. They are diamonds in the rough: while their credit scores lie below bank a 's threshold, their expected types are the highest among those who previously did not borrow.
3. **Welfare Transfer from Good to Great Borrowers** (in terms of observables). As bank a no longer competes for agents with credit scores between η_{ℓ}^B and $\tilde{\eta}_{\ell}^B$, their interest rate rises from $\min\{\rho, r_{\ell}^{B*}\}$ to ρ . However, those with scores above $\tilde{\eta}_{\ell}^B$ see their interest rate fall since the rate bank a must offer to deter bank b is now lower (equation (2.12)).

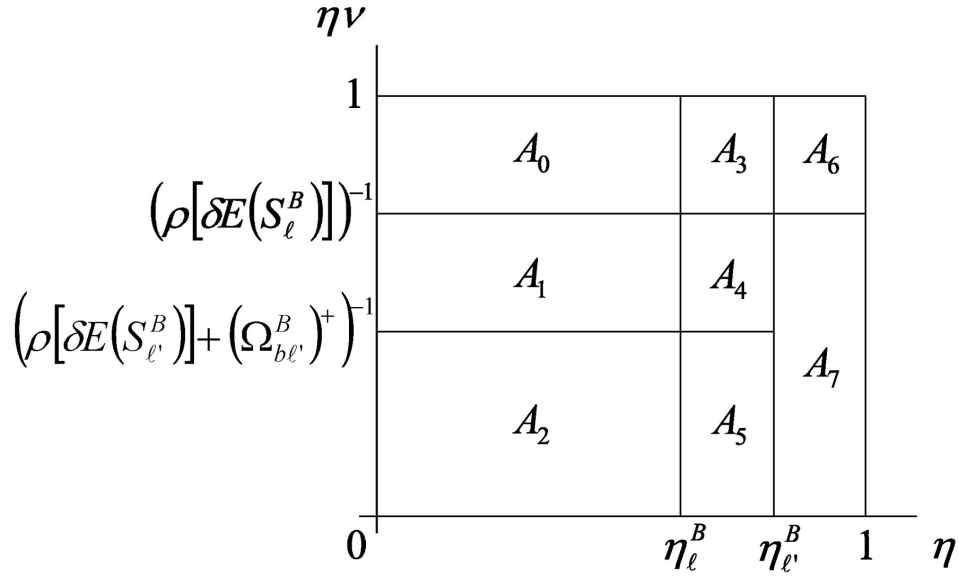


Figure 2.2 Retained Loans Have Lower Expected Default Rates. Two locations ℓ and ℓ' in region B are depicted. The credit rating η appears on the horizontal axis while bank b 's estimate $\eta\nu$ of an applicant's type θ is depicted on the vertical axis. The figure assumes that $E(S_\ell^B) = E(S_{\ell'}^B)$, $\delta\rho E(S_\ell^B) > 1$, $\Omega_{al}^B = \Omega_{al'}^B > 0$, and $\Omega_{b\ell}^B < 0 < \Omega_{b\ell'}^B$. For credit scores below η_ℓ^B , retained loans consist of area A_0 in location ℓ , while securitized loans consist of areas A_0 and A_1 in location ℓ . For credit scores above η_ℓ^B , all loans are securitized in both locations. Hence, for each credit score η , retained loans (if there are any) have a higher expected type $\eta\nu$ than securitized loans.

2.5.3 Higher Securitization Profits for the Remote Bank

Theorem 2.9 implies the following the effects of an increase in bank a 's securitization profits from $\Omega_{a\ell}^B$ to $\widehat{\Omega}_{a\ell}^B$. These are illustrated in Figure 2.4, where $\widehat{\eta}_\ell^B$ denote bank a 's new, lower credit score threshold.

1. **Relatively More Remote Lending.** Bank a lends more, while bank b lends less. In Figure 2.4, a picks up borrowers in areas A_3 through A_5 , while b stops lending to areas A_3 and A_4 and is left with only A_0 and A_1 .
2. **Applicants with High Credit Scores Benefit from More Loans.** The set of borrowers grows to include those with credit scores between bank a 's old and new thresholds (area A_5 in the figure). Among agents who initially did not get loans, these borrowers have the highest credit scores. These agents benefit since their interest rate, $\min\{\rho, r_\ell^{B*}\}$, is lower than the project return ρ .

2.5.4 Efficiency Effects

We next turn to the efficiency effects of securitization. In order for loans to be allocated efficiently *within* each location, a resident of location ℓ in region R must get a loan if and only if her expected project return $\rho\eta\nu E(L_\ell^R)$ exceeds a location-specific threshold c_ℓ^R . In order for the allocation also to be efficient *across* locations and regions, this threshold must not depend on the location ℓ or region R . This is true without securitization, where the threshold c_ℓ^R equals δ^{-1} (Lemma 2.4).

It is useful to restate the condition for within-location efficiency as follows: an agent gets a loan if and only if her expected type $\eta\nu$ exceeds some location-specific threshold \tilde{c}_ℓ^R .³² This holds without securitization, where only agents in areas A_0 and A_3 get loans. However, with securitization it fails, since the threshold is zero if an agent's credit score exceeds η_ℓ^B and $r_\ell^{B*}/\rho > 0$ otherwise.

This discussion reveals two types of inefficiencies that are caused by securitization.

³²In particular, \tilde{c}_ℓ^R equals $c_\ell^R [\rho E(S_\ell^R)]^{-1}$.

1. **Public Information Bias.** Since bank a relies exclusively on public signals to screen agents, there is an inefficient bias towards agents whose public information is strong. In Figure 2.1, agents near the top of area A_2 , who are turned down by both banks, are of higher quality than agents near the bottom of area A_4 , who get loans from bank a .
2. **Securitization Profit Bias.** Efficiency requires that a bank consider only an agent's creditworthiness. However, in equilibrium a bank also prefers agents who are more profitable to securitize. For instance, we can reinterpret Figure 2.3 as comparing two locations in region B , in which bank b 's securitization profits are Ω_{bl}^B and the higher value $\tilde{\Omega}_{bl}^B$, respectively. Agents in the top of A_2 in the former location are turned down, while agents in the bottom of the same area in the latter region receive funding. In Figure 2.4, agents at the top of area A_5 are turned down when bank a 's securitization profits are Ω_{al}^B while agents at the bottom of the same area receive loans when these profits take the higher value $\hat{\Omega}_{al}^B$. In both cases, efficiency requires the opposite.

2.6 Related Literature

While prior models have studied the interaction between a single bank's securitization and lending decisions, ours appears to be the first to study the effect of securitization on lending *competition*. We now discuss the relations between our work and this prior research, as well as related work on security design and on lending competition under adverse selection.

2.6.1 Lending with Securitization

Bubb and Kaufman [11] (BK) study a model with a single bank and a continuum of loan applicants. The bank sees each applicant's credit score. It can also engage in costly screening, which reveals soft information about the applicant. Without securitization, the bank lends to applicants with high scores and rejects those with weak ones. It screens applicants with intermediate scores and lends to them if and only if their soft information is positive. BK then introduce a monopsony loan buyer. The buyer commits to buying a smaller fraction of loans to intermediate borrowers in order to ensure that the bank will still screen them. In

contrast, our model has many small and unorganized security buyers, so such commitment is impossible. Rather, a bank lends remotely in order to have less private information when it issues its security. As a bank cannot discover a remote applicant's soft information, remote lending raises the default risk. In contrast, securitization does not raise defaults in BK.

Hartman-Glaser, Piskorski, and Tchisty [29] study the optimal design of mortgage-backed securities by a lender who can exert costly hidden effort to screen loan applicants. The model takes place in continuous time. Loans default according to a Poisson process. The lender can lower the arrival rate of defaults at a cost. The model is aspatial and features fixed loan terms and a single bank. In contrast, ours is a spatial model with endogenous interest rates and two competing banks. While they study moral hazard, our focus is adverse selection.

Heuson, Passmore, and Sparks [32] (HPS) study a model in which applicants have a continuum of publicly observable default probabilities. A bank chooses whether to lend to an applicant and, if so, whether to securitize the loan. An investor then sets the minimum such probability to accept a loan for securitization. In response, the bank retains the best loans, securitizes intermediate loans, and doesn't lend to the worst borrowers. This mirrors the behavior of a bank towards its local applicants in our model. While HPS study the problem of a single bank under symmetric information, we assume two banks who face asymmetric information at both the lending and securitization stages.

Chemla and Hennessy [13] (CH) also study a model of lending with securitization. A bank can exert costly effort to raise the chance that its loans will have a high return. There are three types of investors. The first group are uninformed risk-averse hedgers for whom the bank's security is a utility-enhancing hedge against future endowment risk. CH offer the example of future home buyers: when the economy booms, few borrowers default on their mortgages, so the security has a high payoff; but the boom also raises house prices, so investors need more money. The security thus hedges against housing market risk. There is also a wealthy, risk-neutral speculator who sees a signal of the asset's type and can exert costly effort to increase the precision of this signal. Finally, there is a continuum of risk-neutral "market makers" with deep pockets.

For some parameters, the model has a pooling equilibrium in which the bank always secu-

ritizes all of its loans. It issues a senior tranche as well as an equity-like mezzanine tranche that is attractive to the hedgers. The hedgers' demand stimulates information acquisition by the speculator, since he can profit from the hedgers' ignorance. The resulting informed speculation increases the correlation of the security price with its true value, which gives the bank an ex ante incentive to screen. This incentive can actually be stronger than in the separating equilibrium in which the bank issues more shares when quality is low. Thus, CH argue that securitization without retention does not necessarily worsen the moral hazard problem, since tranching can lead to informative prices that give the bank an incentive to screen.

Shleifer and Vishny [61] analyze a model in which banks have private information about loan quality (which is either high or low) and must retain a fixed fraction of the loan if they sell it. Loans are sold individually. Security prices are affected by investor sentiment. Since they assume symmetric information with irrational investors, their model bears little relation to ours.

2.6.2 Security Design

Our study is closely related to DeMarzo and Duffie [16] (DD). They study the problem of a risk-neutral issuer who has a fixed portfolio of long term assets. The issuer designs a single security, which consists of a portfolio of assets to securitize and a payoff function: a map from the final value of this portfolio to the security's payoff. The issuer then sees a private signal of the portfolio's value and chooses a proportion of the security to offer for sale to a continuum of uninformed, risk-neutral investors who are more patient than the issuer. There is a unique separating equilibrium. When the issuer's signal is higher, it sells a lower proportion of the security and the market responds with a higher price.

Signalling is costly since the issuer sells less of the security when the gains from trade are greater. For this reason, the issuer's goal at the design stage is to minimize the sensitivity of its security's payoff to its private information. DD show that within the class of monotone, limited-liability securities, this sensitivity is minimized by debt.³³ Intuitively, debt pays its fixed face

³³A security is monotone and limit-liability if its payoff function $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is nondecreasing and satisfies $\varphi(y) \in [0, y]$ for each realization y of the final value of the portfolio of securitized assets.

value when the value of its underlying portfolio exceeds this value. If the debt defaults, it pays the value of its underlying portfolio, which is as close to its face value as limited liability will allow. Hence, the payoff function of debt is as flat as possible within the class of monotone, limited liability payoff functions.³⁴

In DD, the issuer's initial asset portfolio is taken as given. The theoretical contribution of our study is to derive this portfolio as the endogenous result of lending competition. We assume two regional banks who compete for borrowers. The outcome of this competition determines each bank's loan portfolio, which it then securitizes as in DD. Each bank has private information about local applicants at the lending stage. This gives local banks an advantage in competing for loans. Each bank also observes a private signal of its local macroeconomic conditions prior to issuing its security. This creates a lemons problem that favors remote lending.

Since a bank's security can contain a mixture of local and remote loans, a bank's macroeconomic signal contains information about the value of the other bank's security. Hence, the quantity that a bank chooses to sell acts as a signal of the values of both banks' securities. Nevertheless, DD's single-issuer result generalizes: there is a unique separating equilibrium, in which each bank's payoff is the same as in the single-issuer case. Moreover, each bank issues debt. We use this result to derive rich implications for the composition of each bank's loan portfolio.

Like DD, we assume each bank issues at most one security. In contrast, DeMarzo [15] studies the case of a risk-neutral issuer who designs one or more securities based on a finite, exogenous set of assets. The issuer then sees signals of the final values of its assets and chooses how much of each security to sell. DeMarzo shows that pooling the assets before designing the security has a cost and a benefit for the issuer. The cost is information destruction: pooling prevents the issuer from signalling positive information for some securities and negative information for others. The benefit is diversification: if the assets' final values conditional on the signals are not perfectly correlated, then pooling them lowers the risk of security default. Whether pooling is

³⁴Monotonicity is needed since the issuer's signal is noisy. In particular, suppose the bank issues a security that behaves like debt with one exception: its payoff falls slightly in particularly good states. Assume these states have positive probability for intermediate signal values as well. Then this change might lead to a smaller rise in the estimated security payoff as the issuer's signal rises from low to intermediate values. Hence, this security might be even less informationally sensitive than debt.

optimal depends on whether the diversification benefit outweighs the information destruction effect.

Permitting multiple securities would have two effects in our model. First, a bank's profits from securitizing its loans to a given location would depend on which of its various securities it would optimally add the loans to. Second, in the issuance game between the two banks, each bank would choose multiple quantities rather than a single quantity. It seems unlikely that either of these changes would alter our basic results. For simplicity, therefore, we follow DD in restricting each bank to a single security.

Another way banks generate multiple securities is to issue multiple tranches of a single loan portfolio. A bank may also be able to delay designing its security until after it discovers its private information. DeMarzo [15] shows that these practices are equivalent.³⁵ While DD's [16] securitization profit function has a closed form solution, DeMarzo's [15] profit function depends on the solution to a differential equation. This makes it challenging to incorporate into our setting. However, the two functions have some properties in common (DeMarzo [15, Lemmas 5 and 9]), so some of our findings might generalize. This might be an interesting question for future research.

Adverse selection in security issuance was first analyzed by Myers and Majluf [49]. They assume a firm must raise a fixed amount of capital and focus on equity issuance, while briefly considering debt. Nachman and Noe [50] (NN) also assume a fixed amount of capital must be raised but allow for a full set of securities. They give distributional conditions that are sufficient for a firm to issue debt. In their work, the security is designed ex post, while DD assume ex ante design. Axelson [2] reverses the usual informational assumptions: investors are informed while the issuer is not. Like DD and NN, he finds that debt is optimal.

Biais and Mariotti [4] (BM) modify DD in two essential ways. They assume that security buyers have market power and thus earn positive profits. Moreover, they use mechanism design to analyze the optimal trading mechanism, while DD assume that it is a signalling game. BM also find that the optimal security is debt. However, in BM's optimal trading mechanism,

³⁵More precisely, delaying security design until after the (one dimensional) information is revealed is equivalent to issuing an unlimited number of tranches (each of which has a monotone payoff) before the information is revealed.

all issuer types sell 100% of their securities. This contrasts with DD, in which there is some retention.

Boot and Thakor [6] analyze a model in which a firm has various assets that it wishes to sell, and investors can exert costly effort to discover information about these assets' values. There are noise traders, so gathering information can be profitable. Splitting the firm's assets into two securities, one informationally sensitive and the other not, stimulates trade, which gives investors an incentive to discover information about the assets' values. This is profitable for the issuer since it mitigates adverse selection. The results of Chemla and Hennessy [13], discussed above, build on this insight.

Demange and Laroque [17] and Rahi [55] study models in which a risk-averse entrepreneur with a noisy private signal of the value of his projects designs and sells securities. In these papers, unlike DD and ours, the issuer decides how much to issue before observing her private information. The private information only permits the issuer to earn trading profits afterwards.

2.6.3 Lending Competition with Adverse Selection

Our model is also related to prior research on lending competition with adverse selection in the absence of securitization. Perhaps the closest is Hauswald and Marquez [30] (HM). HM assume that a bank's cost of gathering soft information is greater for more distant applicants. This is also true in our model, where the cost is zero for local applicants and infinite for remote ones. Because banks know more about local applicants, they lend at high interest rates to quality local applicants and offer low interest rates to some remote applicants. In HM, the latter effect occurs because other banks - fearing a winner's curse less - compete more aggressively for these remote applicants. In our model, it is because offering a lower interest rate prevents cream-skimming by a remote applicant's local bank. In both models, remote borrowers default more since their lending banks have less information about their credit quality.

In an earlier model, Sharpe [58] assumes that a bank's soft information arises endogenously from its prior loans to applicants. Because of a winner's curse, banks that lack this information do not lend to mature applicants. Analogously, in our model all lending is local if banks cannot securitize. Finally, in Broecker [8], each bank sees a noisy private signal of each loan applicant's

type. Since a bank attracts only those borrowers who are turned down by banks that offer lower rates, a bank that charges a high rate tends to get low quality applicants. Similarly, remote banks in our model charge low interest rates in order to avoid cream-skimming by better informed local banks.

2.7 Conclusions

The model of DeMarzo and Duffie [16] assumes a single issuer who designs a single security. The issuer then sees private information about this security's value and chooses how much to sell. In equilibrium, the issuer varies the amount that it sells in order to signal the security's value. This is costly for the issuer since it must sell less of the security when the gains from trade are higher. In order to minimize these costs, the issuer designs a security that is not very sensitive to its private information.

In DeMarzo and Duffie [16], the issuer's initial portfolio of assets is exogenous. This is an important limitation: in practice, a bank's lending behavior may be influenced by its expected profits from securitizing its resulting loan portfolio. We study this issue in a rich setting in which regional banks first compete for borrowers and then design and issue securities based on their resulting loan portfolios.

As in prior models, we find that securitization expands lending by connecting liquid investors with loan applicants. However, we also find that securitization creates a bias towards remote loans, which can be securitized without contributing to a bank's lemons problem. Moreover, since banks lack soft information about remote applicants, remote borrowers tend to have stronger observables than local borrowers. In addition, banks must offer lower interest rates to remote applicants in order to prevent cream skimming by the applicants' local banks. Thus, remote loans will have lower interest rates than local loans, and securitization strengthens the negative relation between a borrower's public information and the interest rates she pays.

Since banks lack soft information about remote applicants, they do not screen as well when lending remotely. Hence securitization, which stimulates remote lending, raises borrowers' conditional and unconditional default rates. Moreover, in cross section, securitized loans will have higher default rates conditional on observables since banks lower lending standards more

in local areas that are more profitable to securitize. As detailed in section 2.1, all of our predictions are consistent with prior empirical research.

While securitization has the potential to raise social welfare by connecting liquid investors with worthy loan applicants, this is tempered by two inefficiencies. The first is public information bias: since the remote bank relies exclusively on observables, there is an inefficient bias towards applicants with strong observables such as credit scores. This is inefficient as these applicants are favored over creditworthy applicants with weak observables.

The second inefficiency is securitization profit bias. Efficiency requires that only the most creditworthy applicants get loans. However, with securitization banks also prefer applicants who enhance the value of their security. One reason can be that the bank is not well acquainted with the applicant's local macroeconomic environment. In this case, the applicant's loan does not add much to the lemons problem the bank will face in selling its security. Another is that the applicant has a good chance of repaying her loan in bad macroeconomic states. By raising the payout to investors when the security defaults, these borrowers raise the security's value, which translates into greater securitization profits. However, since all participants are risk neutral, it is inefficient to favor these borrowers.

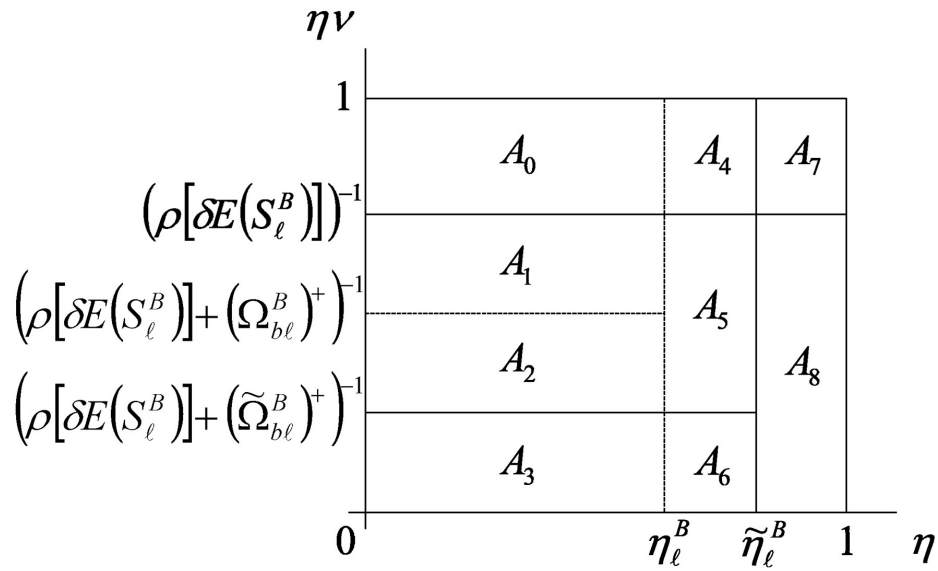


Figure 2.3 Effect of Increase in Bank b 's Securitization Profits from Ω_{bl}^B to $\tilde{\Omega}_{bl}^B$. The conditions of Figure 2.1 are assumed to hold before and after the increase, which raises bank a 's threshold from η_{ℓ}^B to $\tilde{\eta}_{\ell}^B$. Bank a , which initially lent to areas A_4 through A_8 , now only lends to areas A_7 and A_8 and charges a lower interest rate to this group. Bank b adds areas A_2 , A_4 , and A_5 to its initial borrower pool of A_0 and A_1 .

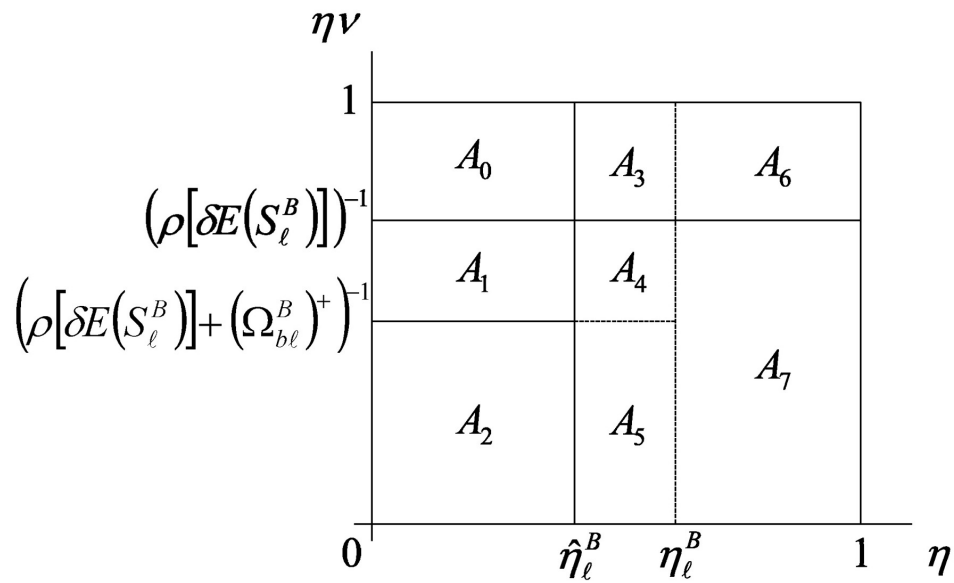


Figure 2.4 Effect of Increase in Bank a 's Securitization Profits from Ω_{al}^B to $\widehat{\Omega}_{al}^B$. The conditions of Figure 2.1 are assumed to hold before and after the increase, which lowers bank a 's threshold from η_{ℓ}^B to $\widehat{\eta}_{\ell}^B$. Bank b ceases to lend to areas A_3 and A_4 and now only lends to areas A_0 and A_1 . Bank a adds areas A_3 through A_5 , and A_5 to its initial borrower pool of A_6 and A_7 . There is no change in the interest rates offered by the two banks.

CHAPTER 3. CREDIT TERMINATION AND TECHNOLOGY BUBBLES

3.1 Introduction

This chapter studies the dynamic interaction between financial intermediaries (banks henceforth) and firms operated by entrepreneurs in a credit cycles model with technology shocks. In this model, a technology shock creates a project with riskier output. The shock on the riskiness of a project mimics a technological innovation. The new project arriving is more attractive to entrepreneurs. But the riskiness of the new project is not observed by banks before lending. After observing a higher default rate, banks deny future loans to entrepreneurs more often in order to affect their choice of projects *ex ante*. The model is used to explain the boom-and-bust of the investment mania in internet technologies, namely the dot-com bubble, in the late 1990s.

Technology shocks could worsen the entrepreneur's incentive problem. More precisely, I assume the set of feasible investment projects changes before and after a technological innovation. Initially, each entrepreneur can invest in one of two projects: a "good" (or "poor") project with a high (respectively, low) expected output. As a standard assumption in a moral hazard model, the entrepreneur needs to pay an additional cost if he chooses the good project. Incentive is needed in order to entice him to invest in the good project. Then the technological innovation gives rise to a third type of project (or a "new" project), which has the same mean output as the poor project but a higher variance. Therefore, it is even more difficult to motivate an entrepreneur to invest in the good project.

For example, before an innovation, suppose that an entrepreneur has two investment opportunities, either to produce food or a movie. Food production, the good project, has a high expected output, while movie-production, the poor project, has a low expected output. Food

production is a boring investment to the entrepreneur, so an incentive is needed for him to invest in this project instead of movie production. After a technology shock, the entrepreneur has a new, even more glamorous option: to invest in a dot-com. This project has the same mean output as movie production, but is even riskier.

Since this type of technology shocks is mean-preserving, there should be no fluctuations without credit market frictions, and entrepreneurs will only choose the good project. But I assume information asymmetry and banks offer loans conditional on their knowledge of the state of the world. Initially, given the optimal loan contract, entrepreneurs should be indifferent between the good and the poor project. Then, the technological innovation creates the new project which is even riskier but more appealing to entrepreneurs (the residual claimant). Biased investment in riskier new projects corresponds to technology bubbles. After knowing that, banks would take actions in order to strengthen the incentive to choose the good project. Banks will cut off the credit lines more often than before. There will be inefficient unemployment of resources, which is essentially what happened at the end of the bubbles. Banks will also transfer incentive rents to entrepreneurs which, due to the zero-profit condition for banks, entail a lower market interest rate for depositors. This causes a decline in the supply of deposits, so there are even fewer loans after the credit termination.

Taking the insight from dynamic contract theory, I assume that a bank, which cannot directly observe an entrepreneur's project choice, has two methods, each associated with a different cost, to motivate the project choice: (i) by giving a (limited liability) rent to the entrepreneur; or (ii) by cutting off the credit line when the entrepreneur defaults. In long-term lending relationships, the distribution of credit histories or "financial capacity" (Gertler [25]) across firms will be an important determinant of aggregate economic activity. Due to low output in the past, the entrepreneur's continuation payoff is close to a threshold in which the bank can no longer motivate him to choose a good project in the future. The credit termination probability rises continuously from zero to one with the optimal rate determined by minimizing the costs to induce the entrepreneur's choice of project. But it also has some social costs, e.g., the economic activities are reduced sharply.

In traditional credit cycle models, shocks affect the mean output of project (e.g., Bernanke

and Gertler [3]; Kiyotaki and Moore [39]). Differently in this model, banks cut off the credit lines and restrict the flow of funds with some probability when the firms have a poor performance in order to turn their attention back to the profitability of the projects. This will generate fluctuations in an economy not only from shocks to the productivity, but also from technology shocks to the variance of the output (the riskiness).

The chapter is structured as follows. I introduce the basic model in the next section. I study the loan contracts and discuss an explicit solution for a special case with two projects and two output levels in Section 3.3. I solve the equilibrium and discuss the comparative static properties in Section 3.4. Finally, I discuss the dot-com bubble which should be considered as one leading example of technology bubbles in the economy in Section 3.5.

3.2 The Model

Time is discrete and the horizon is infinite: $t = 1, 2, \dots$. The economy consists of a sequence of generations, each lives two periods. A newborn agent either becomes an “entrepreneur” or a “lender” with an exogenous probability η or respectively $1 - \eta$. Since the number of births and the number of deaths are equal in each period, the total measure of agents (entrepreneurs and lenders) in this economy is invariant and normalized to one.

A lender is endowed with $h > 0$ units of labor time supplied inelastically in the first period of her life. Each unit of labor time produces one unit of consumption good when she is young. A lender cannot produce when she is old and she has no storage technology. However, she can deposit her goods in a bank. A lender maximizes her life-time utility $U(c_t^y, c_{t+1}^o)$, where, for a lender born at time t , c_t^y and c_{t+1}^o are the consumption when she is young and old.

In each period, an entrepreneur gets access to a set of projects M . For simplicity, suppose that there are two types of projects in the set $M \equiv \{1, 2\}$. Each project produces a stochastic output θ , which takes value from a set $\Theta \equiv \{\theta_1, \theta_2\}$ with $\theta_1 < \theta_2$, in each period of time. Given a project $m \in M$, the output θ is distributed with a density $\pi_i^m = P_\theta(\theta_i|m)$ for $i = 1, 2$ and $\pi_1^m + \pi_2^m = 1$. Project 1 has a smaller probability of producing a high output θ_2 than project 2, or $0 < \pi_2^1 < \pi_2^2 < 1$. Denote $\Delta\pi = \pi_2^2 - \pi_2^1$ the difference of producing a high output. I call project 1 a “poor” project and project 2 a “good” project. Clearly, the distribution satisfies

the monotone likelihood ratio property (MLRP).

Definition 3.1 *The probability $P_\theta(\theta_i|m)$, for $m \in M$, satisfies the MLRP: if $m < m'$, then $\frac{P_\theta(\theta_i|m)}{P_\theta(\theta_i|m')}$ is non-increasing in θ_i .*

Each project $m \in M$ is carried out with two inputs: one unit of capital and an entrepreneurial cost. Entrepreneurs are risk-neutral. For an entrepreneur with a project m , his period utility function at time t is additively separable between the consumption and the entrepreneurial cost and takes the following form: $H(c_t|m) = c_t - v_m$, where $c_t \geq 0$ denotes the entrepreneur's consumption and $v_m \geq 0$ measures the entrepreneurial cost for project m . Normalize the entrepreneurial cost for the poor project to zero $v_1 = 0$; and the cost for the good project is $v_2 = v > 0$. In addition, normalize the entrepreneur's reservation utility to zero in each period. Each entrepreneur is born with zero initial wealth. In order to finance his project, the entrepreneur must borrow money from a bank in the credit market.

Banks arise as institutions of delegated monitoring (Diamond [20]). They gather deposits from lenders and lend to entrepreneurs. While banks do not observe the entrepreneurs' choice of project, they observe the project's output *ex post*. Since the output distribution depends on the type of project, banks can infer entrepreneurs' behavior from the realized output.

There exists a continuum of competitive small banks, each can issue one short-term contract, and one monopoly bank that can issue both short-term and long-term contracts. The short-term contract is standard and only lasts one period. On the other hand, if the long-term contract is used, then, at the end of the first period, the bank have a choice to continue the loan contract or to cut off the credit line and replace the entrepreneur with a new one. The market shares for the small banks and the monopoly bank are $1 - \alpha$ and α , respectively. In addition, the share for the monopoly bank is small enough $\alpha < 1/2$. Hence, after cutting off an entrepreneur's credit line, the monopoly bank is able to find a replacement without additional cost. Each contract also specifies a payment plan conditional on the realized output in order to motivate the entrepreneurs' choice of project.

Finally, let $\bar{\theta}^m = \sum_{i=1,2} \pi_i^m \theta_i$ denote the expected output of project $m \in M$. Throughout the chapter, I assume that it is never optimal to implement the poor project given a gross

interest rate r paid to lenders.¹ Precisely, the good project is socially efficient but the poor project is not, $\bar{\theta}^1 < r < \bar{\theta}^2 - (\Delta\pi)^{-1} \pi_2^2 v$.

3.3 Supply and Demand of Loanable Funds

Terms of the loan contracts will be specified in this section. There are ongoing relationships between entrepreneurs and banks. A bank with full bargaining power lends to an entrepreneur and promises him a certain level of expected utility. At any given time t , the monopoly bank offers a promised utility X_t^y to the young entrepreneurs, and X_t^o to the old to replace the terminated entrepreneurs. After the monopoly bank's offering, the small banks offer a promised utility x_t^y to the young and x_t^o to the old entrepreneurs.

3.3.1 Short-Term Loan Contract

Suppose a bank offers a short-term contract. Omit the time subscript without confusing. The bank will, after announcing which project an entrepreneur should choose, specify a payment plan $\{B_i\}_{i=1,2}$, where B_i is the entrepreneur's payment when the realized output is θ_i . Given the payment plan, the entrepreneur's payoff c_i is $\theta_i - B_i$. Both the bank and the entrepreneur are protected by limited liability. That is, the entrepreneur shall not pay more than the realized output $B_i \leq \theta_i$; and the bank's liability is limited to its initial investment $B_i \geq 0$. The latter one simply makes the entrepreneur's payoff c_i be bounded above by θ_i .

Innes [35] has proved that, with the MLRP and a constraint that the payoff function is non-decreasing in the output, the standard debt contract is optimal in a static model. So, I shall also consider, in each period, the short-term optimal contract takes the form as the standard debt contract: given the face value of the debt (or the gross interest rate charged by the bank) R , the payment $B_i = \theta_i$ if $\theta_i < R$ and $B_i = R$ if $\theta_i \geq R$.

Definition 3.2 *A short-term loan contract is defined by $\{B_i\}_{i=1,2}$ where, for any face value $R \leq \theta_2$, $B_i = \min\{R, \theta_i\}$ for $i = 1, 2$.*

¹Although the gross interest rate r is endogenously determined by the supply and demand of loanable funds, banks and lenders take it as given when they make decisions.

Now, let $m \in M$ be the project suggested by the bank. Assume the bank promises the entrepreneur $x \in [0, \bar{w}]$ to choose m . The promised utility x is greater than zero, otherwise the entrepreneur can walk away; the upper bound \bar{w} comes from the bank's limited liability. The bank's expected profit from the short-term loan $V_S(x)$ is the expected payoff minus the gross interest rate r ,

$$V_S(x) = \max_{\{B_i\}} \sum_{i=1,2} \pi_i^m (B_i - r), \quad (3.1)$$

subject to the individual rationality (IR) constraint (3.2) and the incentive compatibility (IC) constraint (3.3),

$$(IR) \quad \sum_{i=1,2} \pi_i^m c_i - v_m = x, \quad (3.2)$$

$$(IC) \quad m \in \arg \max_{m' \in M} \sum_{i=1,2} \pi_i^{m'} c_i - v_{m'}. \quad (3.3)$$

These two constraints in the incentive problem lead to a threshold utility \underline{w} such that, for any promised utility greater (or less) than the threshold, the good project is (or is not) implementable.

Lemma 3.3 *There is a threshold utility $\underline{w} = (\Delta\pi)^{-1} \pi_1^1 v$ such that:*

1. *for a given promised utility $x \in (\underline{w}, \pi_2^2 \theta_2 - v]$, the good project is implementable with the optimal payment plan $B_1^* = \theta_1$ and $B_2^*(x) = \theta_2 - (\pi_2^2)^{-1} (x + v)$; and*
2. *for a given promised utility $x \in [0, \underline{w}]$, the poor project is implementable with the optimal payment plan $B_1^* = \theta_1$ and $B_2^*(x) = \theta_2 - (\pi_2^1)^{-1} x$.*

Substitute the optimal payment plans from Lemma 3.3 into (3.1). The bank's profit per loan is

$$V_S(x) = \begin{cases} \bar{\theta}^2 - v - r - x, & \text{if } x \geq \underline{w} \\ \bar{\theta}^1 - r - x, & \text{if } x < \underline{w} \end{cases}. \quad (3.4)$$

3.3.2 Long-Term Loan Contract

Suppose a bank can offer a long-term contract. In general, the optimal contract may not be standard debt contracts in a dynamic relationship. However, I restrict attention to those

contracts which have a form similar to debt in each period since debt contracts are commonly observed in practice. Following Stiglitz and Weiss [63], I characterize the long-term relationship between the bank and the entrepreneur as a dynamic debt contract with termination.

Since it is optimal to set $\theta_1 < R \leq \theta_2$, I claim that an entrepreneur defaults when the realized output is θ_1 and he cannot pay the face value of the debt. Define the payment status as an indicator function $\ell(\theta_i \geq R)$, which is one if the entrepreneur pays the face value and zero otherwise. Conditional on the payment status, the bank can choose the entrepreneur's credit record κ . Let the credit record κ take value from a set $K \equiv \{\kappa_0, \kappa_1\}$, where κ_1 is for lending and κ_0 is for termination.

The credit termination condition is thus a distribution of credit record given the payment status. Precisely, when the payment status in the first period is $\ell(\theta_i \geq R)$, the probability of credit record κ_j is $p_{ij} = P_\kappa(\kappa_j | \ell(\theta_i \geq R))$. Since $p_{i0} + p_{i1} = 1$ for all $i \in \{1, 2\}$, to simplify the notation, I shall write the probability of credit line termination p_{i0} as p_i if the payment status is $\ell(\theta_i \geq R)$. And define the credit line termination plan as $\{p_i\}_{i=1,2}$.

In the first period, the payment plan does not depend on the credit record which is determined by the bank at the end of the first period. Let B_i (and $c_i = \theta_i - B_i$) be the entrepreneur's first period payment (respectively, his payoff) when the realized output is θ_i in period one. For any face value $\theta_1 < R \leq \theta_2$ in the first period, I have $B_i = \min\{R, \theta_i\}$ $i = 1, 2$. Following Spear and Wang [62], I shall use the promised utility from the bank to the entrepreneur as the state variable. At the end of period two, the bank will deliver the promised utility x_{ij} to the entrepreneur conditional on the credit record κ_j when the realized output is θ_i in period one.

Definition 3.4 *A long-term loan contract is defined by $\{p_i, B_i, x_{ij}\}$ when the first period output is θ_i and the credit record is κ_j for $i \in \{1, 2\}$ and $j \in \{0, 1\}$.*

Given the suggested project is m , the following problem (3.5) determines the terms of the optimal contract when the bank promises an expected utility exactly equal to X to the entrepreneur at the first period of his life time. The bank's profit per loan $V_L(X)$ from the long run relationship is the expected profit from the first period plus that from the second period

$V(x_{ij})$,

$$V_L(X) = \max_{\{p_i, B_i, x_{ij}\}} \sum_{i=1,2} \pi_i^m \sum_{j=0,1} p_{ij} [(B_i - r) + V(x_{ij})], \quad (3.5)$$

subject to the IR constraint (3.6) and the IC constraint (3.7):

$$(\text{IR}) \sum_{i=1,2} \pi_i^m \sum_{j=0,1} p_{ij} (c_i + x_{ij}) - v_m = X, \quad (3.6)$$

$$(\text{IC}) m \in \arg \max_{m' \in M} \sum_{i=1,2} \pi_i^{m'} \sum_{j=0,1} p_{ij} (c_i + x_{ij}) - v_{m'}. \quad (3.7)$$

I shall solve the optimal long-run loan contract which implements the good project in both periods. It is easy to see that, in the problem (3.5), once the promised utility X is given, the terms of contract in the second period will not affect the first-period choice. So I can solve the contract in the second period as a static loan contract.

First, let the bank's value function in the second period be

$$V(x) = \max \{V_0(x), V_1(x)\}, \quad (3.8)$$

where $V_1(x)$ and $V_0(x)$ are the bank's value function conditional on the credit record κ_1 (lending) and κ_0 (termination), respectively. If the bank lends to the entrepreneur, the second-period contract is equivalent to the static loan contract problem (3.1), $V_1(x) = V_S(x)$. If the bank denies the future loan to the entrepreneur, it will pay the promised utility x to the entrepreneur to end the relationship and look for a replacement in the market. From the Lemma 3.3, to maximize the bank's one-period profit, it promises at least \underline{w} to the new entrepreneur. Thus the expected profit is

$$V_0(x) = \max_y \{V_S(y) | y \geq \underline{w}\} - x. \quad (3.9)$$

The bank will find a replacement after it cuts off an entrepreneur's credit line since there are enough old entrepreneurs who have no investment histories and seek funding in the market.

From (3.4) and (3.9), the profit conditional on credit line termination is

$$V_0(x) = \bar{\theta}^2 - v - r - \underline{w} - x. \quad (3.10)$$

By (3.4), (3.8) and (3.10), the bank's expected profit in the second period is

$$V(x) = \begin{cases} \bar{\theta}^2 - v - r - x, & \text{if } x > \underline{w} \\ \bar{\theta}^2 - v - r - \underline{w} - x, & \text{if } x \leq \underline{w} \end{cases}. \quad (3.11)$$

If $x < \underline{w}$, termination dominates $V_1(x) < V_0(x)$, but if $x \geq \underline{w}$, lending dominates $V_1(x) > V_0(x)$, respectively.

The following Proposition 3.5 from Spear and Wang [62] solves the optimal long-term loan contract. They proved that when the agent's promised utility is too low to support the desired effort, termination occurs as an incentive device in an executive compensation model. The same result holds here that credit line termination is a necessary punishing device if the contract must make the entrepreneur sufficiently poor in the second period.

Proposition 3.5 *The optimal long-term loan contract associated with a promise to deliver expected utility equal to $X \in [0, \bar{w}]$ is:*

1. *if the first period output is low θ_1 , the bank cuts off the credit line with probability $p_1^*(X) = \min \left\{ (2 - \underline{w}^{-1}X)^+, 1 \right\}$,² and the loan plan is*

$$\{B_1^*, x_{10}^*, x_{11}^*\} = \{\theta_1, 0, \underline{w}\};$$

2. *if the first period output is high θ_2 , the bank continues the credit line $p_2^* = 0$, and the loan plan is*

$$\{B_2^*, x_{20}^*, x_{21}^*\} \in \left\{ \begin{array}{l} \{B_2, 0, x_{21}\} : B_2 = R \in [\theta_1, \theta_2] \\ \text{and } x_{21} = R + X + (\Delta\pi)^{-1} (1 - \pi_2^1) v - \theta_2 \end{array} \right\}.$$

The bank's optimal termination policy $\{p_1^*(X), p_2^*\}$ in the terms of loan contract is summarized in the Proposition 3.5. Intuitively, first, the bank will never terminate the entrepreneur's credit line if he has a high output. Second, when the entrepreneur's output is low, the bank will cut off the credit line if the cost of termination is not too high $X \in [0, \underline{w}]$; the bank will cut off the credit line with some positive probability if the termination cost is higher $X \in [\underline{w}, 2\underline{w}]$; and if it is too expensive to terminate $X \in [2\underline{w}, \bar{w}]$, the bank will endure the relationship. In other words, credit termination is a decreasing function of the entrepreneur's initial promised utility X .

²The function $x^+ \equiv \max \{x, 0\}$.

3.3.3 Credit Market Equilibrium

I have shown that the optimal long-term loan contract is characterized by that the entrepreneur with bad outcome will potentially face credit line termination. Now I will define the market equilibrium when banks are competing for both borrowers and depositors (lenders).

Following Stiglitz and Weiss [63], I assume that the first period loans have seniority over the later loans. That is, if the entrepreneur has outstanding obligations, he must repay them before new loans from elsewhere are repaid. Hence, once the credit line is cut off, no banks will finance the defaulting entrepreneur's project. Due to the debt seniority assumption, any entrepreneur who is involved in a long-term relationship cannot sign a new contract with other banks when his credit is terminated. However, any entrepreneur who is in a short-term relationship can sign a new contract with other banks that are looking for one-period investment in the second period because his investment history is private information.

Whenever the bank promises $X_t^o = x_t^o = \underline{w} = (\Delta\pi)^{-1} \pi_2^1 v$, the old entrepreneur accepts the contract. In the credit market, competition will bid up the promised utility so that small banks have zero profit from the young entrepreneur $V_S(x_t^y) = 0$. Thus the promised utility by small banks is $x_t^y(r) = \bar{\theta}^2 - v - r$. Recall the assumption $r < \bar{\theta}^2 - (\Delta\pi)^{-1} \pi_2^2 v$, which implies $\underline{w} < x_t^y(r)$. So small banks have positive return from the old entrepreneur $V_S(x_t^o) > 0$. In order to have zero profit condition, I assume that, if they lend to the old entrepreneurs, the small banks have a cost $\zeta = V_S(x_t^o)$.³

In equilibrium, the young entrepreneur should be indifferent between the short-term and the long-term contracts. Assume that the old entrepreneur has a cost $(1 - \rho)\underline{w}$ to search for a bank and sign a new contract, where $\rho \in (0, 1)$. The promised utility from the monopoly bank should equal to the utility from the short-term loans $X_t^y = x_t^y + \rho\underline{w}$. For the long-term loan contract, the promised utility is $X_t^y(r) = \bar{\theta}^2 - v - r + \rho\underline{w}$.

All entrepreneurs except those terminated require one unit of funds. To measure the demand of investment funds, I can simply measure the entrepreneurs terminated in the second period. Since the monopoly bank writes a large number of loan contracts with the entrepreneurs, the

³For example, due to the debt seniority, the entrepreneur who is involved in a long-term relationship and is cut off credit lines may seek funding to pay the debt. The competitive small banks have to screen those firms.

measure, by the law of large numbers, is the measure of entrepreneurs taking the long-term contract $\frac{1}{2}\alpha\eta$, times the probability of termination $\pi_1^2 p_1^*(X_t^y(r))$, or

$$\mu(r|\underline{w}) = \frac{1}{2}\alpha\eta\pi_1^2 \min \left\{ \left(2 - \rho - \underline{w}^{-1} \left(\bar{\theta}^2 - v - r \right) \right)^+, 1 \right\}. \quad (3.12)$$

The measure $\mu(r|\underline{w})$ is weakly increasing in the threshold utility \underline{w} .

Lenders' consumption smoothing preference determines the supply of the loanable funds. In each period t , taking the market gross interest rate r_t as given, lenders solve the following inter-temporal utility maximization problem,

$$U(c_t^y, c_{t+1}^o) = \max_{s_t} u(c_t^y) + E[c_{t+1}^o],$$

subject to $c_t^y + s_t = h$ and $c_{t+1}^o = r_t s_t$. Here $E[\cdot]$ denotes the expectation. And $s_t \in [0, h]$ is the representative lender's savings in the first period. The utility function $u(\cdot)$ takes the usual concave form: $u'(\cdot) > 0$ and $u''(\cdot) < 0$, where $u'(\cdot)$ and $u''(\cdot)$ denote the first and second order derivatives with respect to the consumption. Lenders are risk-neutral with respect to the consumption when they are old, and the discount factor is one for simplicity. So lenders save when they are young and consume the savings when they are old. Under the assumption that an interior solution exists, $0 < s_t < h$, the following first-order condition implicitly defines the savings $u'(h - s_t) = r_t$. From the first-order condition, I can also solve, in equilibrium, the savings as a function of the market interest rate,

$$s_t = s(r_t) = h - (u')^{-1}(r_t). \quad (3.13)$$

Finally, to close the model, in equilibrium, the loanable funds market clearing condition requires that the loans are equal to the savings. Intuitively, the young entrepreneurs are *ex ante* identical and thus, without credit rationing, they will receive the bank loan. But the old entrepreneurs may have different investment histories after signing the long-term contracts. If the old entrepreneurs defaulted in the first period, their projects may be terminated at the end of the first period. The total demand for investment funds is $\eta - \mu(r_t|\underline{w})$ and the market clearing condition is

$$\frac{1}{2}(1 - \eta) s(r_t) = \eta - \mu(r_t|\underline{w}). \quad (3.14)$$

Definition 3.6 Given the termination policy $\{p_i^*\}_{i=\{1,2\}}$ from the banks' optimal loan contracts (Proposition 3.5), the credit market equilibrium is a pair of saving and gross interest rate $\{s^*, r^*\}$ solved from equations (3.12), (3.13) and (3.14).

3.4 Shocks and Technology Bubbles

In this section, I introduce a technology shock. The mania stage of a technology bubble is on the off-equilibrium path where banks do not adjust loan contracts due to information asymmetry. After more and more entrepreneurs default the loan repayment, banks will restrict credit. That is, entrepreneurs with low output are terminated with a higher probability which is related to the crash of a technology bubble.

3.4.1 Technology Shocks

The special feature of technology shocks is they will not change the productivity but the riskiness of the economy. Importantly, I assume the state at time t is only known to the entrepreneurs who operate the projects. Banks can infer the state at time t only from the default rate of the economy at the beginning of time $t + 1$. Let ω_t , taking value from the finite set $\Omega \equiv \{\omega_1, \omega_2\}$, denote the state of the economy at time t . In both a normal state ($\omega_t = \omega_1$) or a bubble state ($\omega_t = \omega_2$), the set of feasible investment projects is $M_{\{\omega_t\}} = \{1, 2\}$. And the project output takes value from $\{\theta_1(\omega_t), \theta_2(\omega_t)\}$ with distribution $\{\pi_i^m(\omega_t), \pi_i^m(\omega_t)\}$ for $m \in M_{\{\omega_t\}}$.

Assume the good project is the same in both states. In the bubble state, project 1 is even riskier. I call this project the “new” project. Specifically, assume the project 1 in both states has the same salvage value after defaulting $\theta_1(\omega_t) = \theta_1$ for $\omega_t \in \Omega$, and the same expected return

$$\sum_{i=1,2} \pi_i^1(\omega_1) \theta_i(\omega_1) = \sum_{i=1,2} \pi_i^1(\omega_2) \theta_i(\omega_2).$$

The expected output of the new project is mean preserving to the poor project. However, if the entrepreneur chooses the new project, he has a lower probability $\pi_2^1(\omega_2) < \pi_2^1(\omega_1)$ to produce an even higher output $\theta_2(\omega_2) > \theta_2(\omega_1)$ in the bubble state.

Further, in order to be consistent with the information structure in the economy, I assume banks do not know the state at time t from the realized output.⁴ To do this, I assume banks cannot distinguish the good outcomes in the two states: $\theta_2(\omega_1)$ and $\theta_2(\omega_2)$. This is a restrictive but reasonable assumption because, when the project succeeds, the debt is fully repaid and banks get the same loan interest rate in both states ω_1 and ω_2 . And, when the entrepreneur defaults, the bank gets the same salvage value $\theta_1(\omega_t) = \theta_1$ before and after the technology shock. However, when the entrepreneurs choose the new project in the bubble state, the probability of default is higher than the probabilities when they choose the good project $\pi_1^1(\omega_2) > \pi_1^2(\omega_1)$, or the poor project $\pi_1^1(\omega_2) > \pi_1^1(\omega_1)$ in the normal state. Therefore, banks can infer the state by the default rate in the economy.

3.4.2 Technology Bubbles

Since the preference, average productivity, and population are identical in each period, the mean preserving technology shocks will not produce fluctuations in the absence of credit market frictions. However, with credit market frictions (moral hazard and limited liability), it is even more difficult to induce the entrepreneur to choose the good project with the technology shocks.

A comparative static analysis shows how aggregate variables depend on the state. Fix, initially, the economy in state ω_1 with an equilibrium $\{s^*(\omega_1), r^*(\omega_1)\}$ until period $t = T$. After the technology shock at $t = T$, the model economy changes to state ω_2 . Then in a later period $t' \geq T + 1$, the banks infer the bubble from a high proportion of defaults, and they restrict the credit to restore the new equilibrium $\{s^*(\omega_2), r^*(\omega_2)\}$. By equations (3.12), (3.13) and (3.14), I can solve equilibria in both states. I restrict on the case where

$$(2 - \rho)^{-1} (\bar{\theta}^2 - v - r) < \underline{w} < (1 - \rho)^{-1} (\bar{\theta}^2 - v - r).^5$$

Proposition 3.7 gives the predictions of the model economy with the technology shock.

⁴Mirrlees [45] has shown that, if the support of distribution varies with different projects, the first-best can be achieved when the support is observable.

⁵If $\underline{w} \leq (2 - \rho)^{-1} (\bar{\theta}^2 - \psi - r)$ or $\underline{w} \geq (1 - \rho)^{-1} (\bar{\theta}^2 - \psi - r)$, then $\mu(x)$ equals 0 or $\frac{1}{2}\eta\alpha\pi_1^2$ respectively. In these two scenarios, there is no change in the rate of credit termination. We rule out these two scenarios and focus on the more interesting case.

Proposition 3.7 *With the technology shock, the economy changes from state ω_1 to state ω_2 .*

The model predicts:

1. *If the banks cannot response immediately to the technology shock, entrepreneurs will switch to the glamorous new project.*

2. *After the shock is disclosed to banks, they restrict the credit to restore the good project.*

But,

(a) *there is an inefficient unemployment of resources after the shock; and*

(b) *the market interest rate and savings decrease, $r^*(\omega_2) < r^*(\omega_1)$ and $s^*(\omega_2) < s^*(\omega_1)$.*

Intuitively, after the technology shock, the banks, due to the information structure, cannot response to it immediately. However, the entrepreneurs observe it and choose the glamorous new project since it is intrinsically appealing to them. I interpret that entrepreneurs switch to the glamorous new project as a technology bubble which is on the off-equilibrium path. And after the banks observe the high default rate and restrict the credit, the bubble bursts and the good project is restored. In this model, firms are driven out of the credit market by the increased rate of credit line termination. The market interest rate falls because, in order to promise a higher utility to entrepreneurs, banks must lower the cost of the funds to keep a zero profit given that entrepreneurs are *ex ante* identical.

3.5 Credit Termination and the Dot-Com Bubble - An Example

The dot-com bubble, covering roughly from 1998 to 2001, is one of the most biggest technology bubbles in terms of scope. During the booming period, although the U.S. stock market measured by the S&P 500 index was in an up trend, the technology heavy NASDAQ Composite index was growing even faster. Previous literature focuses on the limits of arbitrage in the stock market, e.g., the effect of short sale restriction (Ofek and Richardson [52]) and strategic bubble riding of institutional investors (Brunnermeier and Nagel [9]).

This chapter studies the dot-com bubble from a different angle. Consistent with the model, at the dot-com bubble period, the internet sector had nothing unusual that could explain

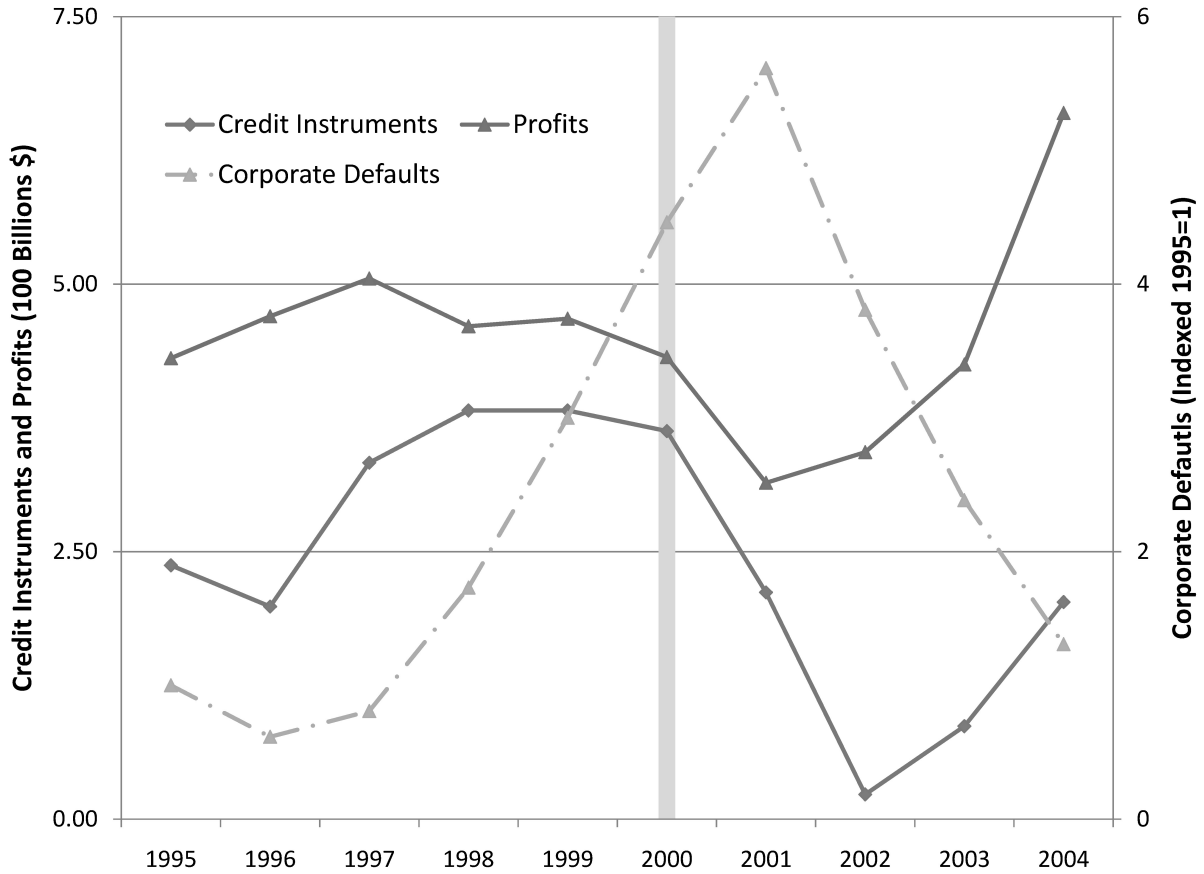


Figure 3.1 Credit, Profit, and Default before and after the Dot-Com Bubble Period. Data Source: Credit instruments are from Fed's Flow of Funds (100 Billions \$), and corporate defaults are from Standard and Poor's (Indexed, 1995=1).

the booming. Economy-level and sector-level research on the relationship between technology and productivity found little supporting evidence of productivity improvement (see Gordon [28]). However, banks might misunderstand the new incremental technological changes due to information asymmetry. Figure 3.1 records that, from 1998 to 2000, there was a large increase in the firms' net liabilities.

In the dot-com bubble period, entrepreneurs were attracted by online business ideas: online stores, delivery services, banking, etc. In fact, there was no significant sign of profit increasing during the dot-com bubble period (Figure 3.1). They took the risk and invested in the dot-com sector because they might take all the shares of the market and gain profits if others defaulted. Hence, they were looking for more funds to sustain these projects. The booming, peaked in March 2000, lasted more than two years and made investors believe that it was a “new economy paradigm”. Investors were convinced the productivity improvement in the future.

The default rate increased due to the higher proportion of low return (Figure 3.1). Brynjolfsson and Hitt [10] suggest a positive relationship between the information technology investment and productivity, but also a great deal of individual variation in firms' success with information technology. With the increasing default rate, banks realized the higher risk in the dot-com sector. Hence the investment funds dried up. Observing this, investors reduced their positions in technology stocks and the crash period followed.

3.6 Conclusion

Beliefs that some technological innovations (e.g., internal combustion engine, electric motor, internet, green energy, etc.) change the productivity make the investors eager to catch up the technological wave before they fully understand the innovations. Some of them did make fundamental transformations in the economy, while others were just incremental technological changes. In the latter case, investment manias may appear and cause inefficiency.

In this chapter, I argue that technology shocks may be amplified and thus lead to severe economic fluctuations with credit market frictions. Bank loans are signals about where to allocate the real resources in the economy. The misunderstanding of the investment opportunity leads to a bubble because the stock market believes banks may have private good news about

the new technology. The stock market may over-react to the signals. An interesting example further supports the irrationality of investors is the overpricing of Palm-3Com discussed in Lamont and Thaler [42].

I also study contingency loan contracts in which the bad outcome may lead to an end of the lending relationships. Banks cannot observe the entrepreneurs' investment behavior. So credit termination is used as an incentive device to affect an entrepreneur's choice of project *ex ante*. Stiglitz and Weiss [63] pointed out "banks often deny future loans to defaulters rather than raising the interest rate that a defaulter would have to pay". This is connected to the withdrawal of credit after the burst of the technology bubbles. Therefore, proper designs of loan contracts and regulations are required to reduce the fluctuations and inefficiency.

APPENDIX A. ADDITIONAL MATERIAL FOR CHAPTER 1

Proofs

Optimal Contracts

Proof of Lemma 1.1: First, conditional on the investment in period 1, the bank can choose to monitor all borrowers or to monitor borrowers based on the first period outcome. If the second period profit from the former method is less than that from the latter one,

$$\delta\mu_h (p_H R_2 - D) - (\mu_h + \mu_l) m < \delta V_2^0 (R_2; \mu) - C(m, \mu), \quad (\text{A.1})$$

the bank will choose the latter one. The condition (A.1) is equivalent to

$$(\tilde{\mu}_h + \tilde{\mu}_l) m + \delta\tilde{\mu}_l (p_L R_2 - D) > 0.$$

So when the monitoring cost is large enough $m > \hat{m} \equiv \delta (\tilde{\mu}_h + \tilde{\mu}_l)^{-1} \tilde{\mu}_l D$, it is more efficient to monitor the borrowers based on the first period outcome.

Due to limited liability, the interest rate conditional on default is zero: $R_1^{0*}(0) = R_2^{0*}(0) = 0$. Any optimal contract $\{R_1^{0*}, R_2^{0*}\}$ must satisfy the incentive compatibility (1.5) and the individual rationality (1.4) constraints. The individual rationality constraint (1.4) is binding to maximize the bank's expected profit. Hence, I have

$$R_2 = c_1 - R_1, \text{ where } c_1 \equiv 2\theta - \frac{v_0}{p_H}. \quad (\text{A.2})$$

That is, the bank must increase an equal unit of interest rate in period 2 to reduce the interest rate in period 1.

To solve the bank's problem and find the optimal contract, let us put (A.2) into the objective

function (RL):

$$\begin{aligned}
& V_1^0(R_1; \mu) - C(m, \mu) + \delta V_2^0(R_2; \mu) \\
&= \mu_h v_H(R_1) + \mu_l v_L(R_1) - [\mu_h(1 - p_H) + \mu_l(1 - p_L)]m \\
&\quad + \delta [\mu_h v_H(R_2) + \tilde{\mu}_l v_L(R_2)] \\
&= [\tilde{\mu}_h(1 - \delta) + \tilde{\mu}_l(1 - \delta p_L)]R_1 + \delta(\tilde{\mu}_h + \tilde{\mu}_l p_L)c_1 \\
&\quad - [\mu_h(1 - p_H) + \mu_l(1 - p_L)]m - [\mu_h + \mu_l + \delta(\mu_h + \tilde{\mu}_l)]D.
\end{aligned}$$

The expected profit is thus the gain from loans minus the cost to monitor the borrowers and the cost to collect funds. Since $p_L < 1$ and $\delta < 1$, I have

$$\tilde{\mu}_h(1 - \delta) + \tilde{\mu}_l(1 - \delta p_L) > 0.$$

Hence, it is optimal to set $R_1^{0*} = \theta$ to maximize (RL). Thus I have $R_2^{0*} = \theta - v_0/p_H$ from (A.2), where $R_2^{0*} > 0$ since $v_0 < p_H^2 \theta < p_H \theta$.

Finally, I must check that the optimal contract $\{R_1^{0*}, R_2^{0*}\}$ satisfies the incentive constraint (1.5). This constraint, which holds as an inequality, gives us the following inequality between R_1 and R_2 :

$$R_2 \leq c_2 - \frac{p_H - p_L}{p_H - p_L^2} R_1, \text{ where } c_2 \equiv \left(1 + \frac{p_H - p_L}{p_H - p_L^2}\right) \theta - \frac{b}{p_H - p_L^2}. \quad (\text{A.3})$$

When the private benefit is not too large $b \leq (p_H^2 - p_L^2)v_0/p_H < (p_H - p_L^2)v_0/p_H$, I have

$$R_2^{0*} = \theta - \frac{v_0}{p_H} \leq \theta - \frac{b}{(p_H - p_L^2)} = c_2 - \frac{p_H - p_L}{p_H - p_L^2} R_1^{0*}.$$

Thus the optimal contract $\{R_1^{0*}, R_2^{0*}\}$ satisfies the inequality (A.3). Q.E.D.

Proof of Lemma 1.2: The proof is similar to the previous Lemma 1.1. Due to limited liability, the interest rate conditional on default is zero: $R_1^{1*}(0) = R_2^{1*}(0) = 0$. Any optimal contract $\{R_1^{1*}, R_2^{1*}\}$ must satisfy the incentive compatibility (1.9) and the individual rationality (1.8) constraints. Moreover, the individual rationality constraint (1.8) is binding since the bank wants to reduce the borrower's payoff to maximize its expected profit. Hence, I have

$$R_2 = c_3 - \frac{1}{p_H} R_1, \text{ where } c_3 \equiv \left(1 + \frac{1}{p_H}\right) \theta - \frac{v_0}{p_H^2}. \quad (\text{A.4})$$

That is, in order to give the borrower the promised utility, the bank must increase $1/p_H$ interest rate in period 2 to reduce one unit of interest rate in period 1.

To solve the bank's problem and give the optimal contract, let us put (A.4) into the objective function (TL):

$$\begin{aligned}
& V_1^1(R_1; \mu) + \rho(\gamma) V_2^1(R_2; \mu) \\
&= \mu_h v_H(R_1) + \mu_l v_L(R_1) + \rho(\gamma) [\tilde{\mu}_h v_H(R_2) + \tilde{\mu}_l v_L(R_2)] \\
&= \left[\tilde{\mu}_h (1 - \rho(\gamma)) + \tilde{\mu}_l \left(1 - \rho(\gamma) \frac{p_L}{p_H} \right) \right] R_1 \\
&\quad + \rho(\gamma) (\tilde{\mu}_h p_H + \tilde{\mu}_l p_L) c_3 - [\mu_h + \mu_l + \rho(\gamma) (\tilde{\mu}_h + \tilde{\mu}_l)] D.
\end{aligned}$$

The expected profit is thus the gain from loans minus the cost of collecting funds. Since $p_H > p_L$ and $\rho(\gamma) < 1$, I have

$$\tilde{\mu}_h (1 - \rho(\gamma)) + \tilde{\mu}_l \left(1 - \rho(\gamma) \frac{p_L}{p_H} \right) > 0.$$

Hence, it is optimal to set $R_1^{1*} = \theta$ to maximize (TL). Thus I have $R_2^{1*} = \theta - v_0/p_H^2$ from (A.4).

Under the assumption on the promised utility $v_0 < p_H^2 \theta$, I have $R_2^{1*} > 0$ when $R_1^{1*} = \theta$.

Finally, I must check that the optimal contract $\{R_1^{1*}, R_2^{1*}\}$ satisfies the incentive compatibility constraint (1.9). This constraint, which holds as an inequality, gives us the following inequality between R_1 and R_2 :

$$R_2 \leq c_4 - \frac{1}{p_H + p_L} R_1, \text{ where } c_4 \equiv \left(1 + \frac{1}{p_H + p_L} \right) \theta - \frac{b}{p_H^2 - p_L^2}. \quad (\text{A.5})$$

Under the assumption on the private benefit $b \leq (p_H^2 - p_L^2) v_0/p_H < (p_H^2 - p_L^2) v_0/p_H^2$, I have

$$R_2^{1*} = \theta - \frac{v_0}{p_H^2} \leq \theta - \frac{b}{(p_H^2 - p_L^2)} = c_4 - \frac{1}{p_H + p_L} R_1^{1*}.$$

Thus the optimal contract $\{R_1^{1*}, R_2^{1*}\}$ satisfies the inequality (A.5). Q.E.D.

Properties of the Profit Gain (B1-B3)

From equations (1.11) and (1.12), the bank's profit from transactional lending is

$$U(1, \lambda, \gamma) = \mu_h v_H(R_1^{1*}) + \pi_l^1(\lambda) v_L(R_1^{1*}) + \rho(\gamma) [\tilde{\mu}_h v_H(R_2^{1*}) + \tilde{\pi}_l^1(\lambda) v_L(R_2^{1*})],$$

and the profit from relationship lending is

$$U(0, \lambda, \gamma) = \mu_h [v_H(R_1^{0*}) - (1 - p_H)m + \delta v_H(R_2^{0*})].$$

In addition, since the first period interest rates are the same $R_1^{1*} = R_1^{0*} = \theta$, the first period payoffs from the two technologies are the same $v_H(R_1^{1*}) = v_H(R_1^{0*})$. Hence, the profit gain $\omega(\lambda, \gamma) = U(1, \lambda, \gamma) - U(0, \lambda, \gamma)$ is

$$\begin{aligned} \omega(\lambda, \gamma) &= \pi_l^1(\lambda) v_L(R_1^{1*}) + \rho(\gamma) [\tilde{\mu}_h v_H(R_2^{1*}) + \tilde{\pi}_l^1(\lambda) v_L(R_2^{1*})] \\ &\quad + \mu_h [(1 - p_H)m - \delta v_H(R_2^{0*})]. \end{aligned} \quad (\text{A.6})$$

First, given the definition of $\rho(\gamma)$ in (1.6), the derivative of the profit gain (A.6) with respect to the state of liquidity γ gives us

$$\frac{\partial \omega(\lambda, \gamma)}{\partial \gamma} = \begin{cases} V_2^1(R_2^{1*}; \pi^1(\lambda)), & \text{if } \gamma \in (\delta, 1) \\ 0, & \text{otherwise} \end{cases}. \quad (\text{A.7})$$

I need to show the payoff in the second period is non-negative:

$$V_2^1(R_2^{1*}; \pi^1(\lambda)) = \tilde{\mu}_h v_H(R_2^{1*}) + \tilde{\pi}_l^1(\lambda) v_L(R_2^{1*}) \geq 0.$$

Since the measure of the type- l borrowers $\tilde{\pi}_l^1(\lambda)$ takes the maximum when the proportion λ is 0: $\tilde{\pi}_l^1(\lambda) = (\lambda \ell_1 + \ell_0)^{-1} \tilde{\mu}_l \leq \tilde{\pi}_l^1(0) = \ell_0^{-1} \tilde{\mu}_l$ and the payoff from the type- L project is negative $v_L(R_2^{1*}) < 0$, I have

$$\tilde{\mu}_h v_H(R_2^{1*}) + \tilde{\pi}_l^1(\lambda) v_L(R_2^{1*}) \geq \tilde{\mu}_h v_H(R_2^{1*}) + \ell_0^{-1} \tilde{\mu}_l v_L(R_2^{1*}).$$

Therefore, it is enough to show

$$0 < \tilde{\mu}_h v_H(R_2^{1*}) + \ell_0^{-1} \tilde{\mu}_l v_L(R_2^{1*}), \text{ or } \ell_1 < \frac{V_2^1(R_2^{1*}; \mu)}{\tilde{\mu}_h v_H(R_2^{1*})}. \quad (\text{A.8})$$

Since the payoffs from the type- L projects $v_L(R_2^{1*})$ and $v_L(R_1^{1*})$ are negative, the following inequality holds

$$-\delta \tilde{\mu}_l v_L(R_2^{1*}) - \mu_l v_L(R_1^{1*}) > 0,$$

which implies

$$(1 - \delta) \tilde{\mu}_h v_H(R_2^{1*}) < (1 - \delta) V_2^1(R_2^{1*}; \mu) - [\mu_l v_L(R_1^{1*}) + \tilde{\mu}_l v_L(R_2^{1*})],$$

which further implies the following inequality

$$\frac{V_2^1(R_2^{1*}; \mu)}{\tilde{\mu}_h v_H(R_2^{1*})} > \frac{(1 - \delta) V_2^1(R_2^{1*}; \mu)}{(1 - \delta) V_2^1(R_2^{1*}; \mu) - [\mu_l v_L(R_1^{1*}) + \tilde{\mu}_l v_L(R_2^{1*})]}.$$

Moreover, by the definition (1.10), I have

$$\hat{\ell}_1 \equiv \frac{(1 - \delta) V_2^1(R_2^{1*}; \mu)}{(1 - \delta) V_2^1(R_2^{1*}; \mu) - [\mu_l v_L(R_1^{1*}) + \tilde{\mu}_l v_L(R_2^{1*})]}.$$

Since I assume $\ell_1 \leq \hat{\ell}_1$, the inequality (A.8) holds, hence the payoff in the second period is non-negative. Therefore, the partial derivative is non-negative $\partial\omega(\lambda, \gamma)/\partial\gamma \geq 0$ for all $\lambda \in [0, 1]$. This proves the state monotonicity (B1).

Second, take a derivative of the profit gain $\omega(\lambda, x)$ with respect to λ :

$$\frac{\partial\omega(\lambda, \gamma)}{\partial\lambda} = -\frac{\ell_1 \mu_l v_L(R_1^{1*}) + \rho(\gamma) \ell_1 \tilde{\mu}_l v_L(R_2^{1*})}{(\lambda \ell_1 + \ell_0)^2}. \quad (\text{A.9})$$

Since both $v_L(R_1^{1*})$ and $v_L(R_2^{1*})$ are negative, the profit gain $\omega(\lambda, x)$ is increasing in the proportion λ : $\partial\omega(\lambda, \gamma)/\partial\lambda > 0$ for any given state of liquidity $\gamma \in \mathbf{R}^{++}$. This proves the strategic complementarities (B2).

Finally, I need to show the upper and lower dominance regions are feasible. Before that, I need to check the interval (\underline{m}, \bar{m}) is non-empty. Recall the definition of the payoff

$$\begin{aligned} V_2^1(R_2^{1*}; \pi^1(0)) &= \tilde{\mu}_h v_H(R_2^{1*}) + \ell_0^{-1} \tilde{\mu}_l v_L(R_2^{1*}) \\ &= V_2^1(R_2^{1*}; \mu) + \ell_0^{-1} \ell_1 \tilde{\mu}_l v_L(R_2^{1*}). \end{aligned}$$

Then, from the definition (1.10), I have the following equivalent inequalities

$$\begin{aligned} \frac{\ell_1}{\ell_0} &< \frac{(1 - \delta) V_2^1(R_2^{1*}; \mu)}{-\mu_l v_L(R_1^{1*}) - \tilde{\mu}_l v_L(R_2^{1*})} \\ \iff &(1 - \delta) V_2^1(R_2^{1*}; \mu) > -\ell_0^{-1} \ell_1 \mu_l v_L(R_1^{1*}) - \ell_0^{-1} \ell_1 \tilde{\mu}_l v_L(R_2^{1*}) \\ \iff &-\mu_l v_L(R_1^{1*}) - \delta V_2^1(R_2^{1*}; \mu) > -\ell_0^{-1} \mu_l v_L(R_1^{1*}) - V_2^1(R_2^{1*}; \pi^1(0)). \end{aligned}$$

Comparing with the definitions (1.14) and (1.15), I have the result (plus $\delta \mu_h v_H(R_2^{0*})$ and divide by $\mu_h(1 - p_H)$).

From strategic complementarities (B2), the profit gain is non-decreasing in the proportion λ . When the proportion λ is 1, the profit gain takes the maximum: $\omega(1, \gamma) \geq \omega(\lambda, \gamma)$ for all

$\lambda \leq 1$. In addition, from equation (A.6), I have the following equation

$$\omega(\lambda, \gamma)|_{\lambda=1, \gamma=\delta} = \begin{bmatrix} \mu_l v_L(R_1^{1*}) + \delta V_2^1(R_2^{1*}; \mu) \\ + \mu_h [(1 - p_H) m - \delta v_H(R_2^{0*})] \end{bmatrix}. \quad (\text{A.10})$$

Since the monitoring cost is low enough $m < \bar{m}$, by the definition (1.15), I have

$$\begin{aligned} m &< \frac{\delta \mu_h v_H(R_2^{0*}) - \mu_l v_L(R_1^{1*}) - \delta V_2^1(R_2^{1*}; \mu)}{\mu_h (1 - p_H)} \equiv \bar{m} \\ &\iff \mu_l v_L(R_1^{1*}) + \delta V_2^1(R_2^{1*}; \mu) + \mu_h [(1 - p_H) m - \delta v_H(R_2^{0*})] < 0. \end{aligned}$$

Hence the profit gain is negative given the proportion $\lambda = 1$ and the state of liquidity $\gamma = \delta$: $\omega(\lambda, \gamma)|_{\lambda=1, \gamma=\delta} < 0$. Then, there exists $\underline{\gamma} > \delta$ such that $\omega(1, \underline{\gamma}) < 0$ since the profit gain $\omega(\lambda, \gamma)$ is continuous. Hence, the profit gain is negative $\omega(1, \gamma) < 0$ for all $\gamma \in [\delta, \underline{\gamma}]$ since the profit gain is non-decreasing in γ by the state monotonicity (B1). So there exists a lower dominance region $[\delta, \underline{\gamma}]$ since $\omega(\lambda, \gamma) \leq \omega(1, \gamma) < 0$. Thus the lower region exists.

Similarly, from the property (B2) again, when the proportion $\lambda = 0$, the profit gain takes the minimum: $\omega(0, \gamma) \leq \omega(\lambda, \gamma)$ for all $\lambda \geq 0$. In addition, from equation (A.6), I have the following equation

$$\omega(\lambda, \gamma)|_{\lambda=0, \gamma=1} = \begin{bmatrix} \ell_0^{-1} \mu_l v_L(R_1^{1*}) + V_2^1(R_2^{1*}; \pi^1(0)) \\ + \mu_h [(1 - p_H) m - \delta v_H(R_2^{0*})] \end{bmatrix}. \quad (\text{A.11})$$

Since that the monitoring cost is large enough $m > \underline{m}$, by the definition (1.14), I have

$$\begin{aligned} m &> \frac{\delta \mu_h v_H(R_2^{0*}) - \ell_0^{-1} \mu_l v_L(R_1^{1*}) - V_2^1(R_2^{1*}; \pi^1(0))}{\mu_h (1 - p_H)} \equiv \underline{m} \\ &\iff \ell_0^{-1} \mu_l v_L(R_1^{1*}) + V_2^1(R_2^{1*}; \pi^1(0)) + \mu_h [(1 - p_H) m - \delta v_H(R_2^{0*})] > 0. \end{aligned}$$

Hence the profit gain is positive given the proportion $\lambda = 0$ and the state of liquidity $\gamma = 1$: $\omega(\lambda, \gamma)|_{\lambda=0, \gamma=1} > 0$. Then, there exists $\bar{\gamma} < 1$ such that $\omega(0, \bar{\gamma}) > 0$ since the profit gain $\omega(\lambda, \gamma)$ is continuous. Hence, I have the profit gain $\omega(0, \bar{\gamma}) > 0$ for all $\gamma \in [\bar{\gamma}, 1]$ since the profit gain is non-decreasing in γ by the state monotonicity (B1). So there exists an upper dominance region $[\bar{\gamma}, 1]$ since $\omega(\lambda, \gamma) \geq \omega(0, \gamma) > 0$. Thus the upper region exists.

Unique Equilibrium

Proof of Theorem 1.5: Consider the bank that saw a private signal x . By Bayes's Rule, its posterior probability $P^\sigma(\gamma|x)$ over the state of liquidity γ at any point γ_0 is proportional

to the product of the density of private signal x conditional on γ_0 and the prior density of the state of liquidity γ at the point γ_0 :

$$P^\sigma(\gamma_0|x) = \frac{\phi\left(\frac{1}{\sigma}\ln(x/\gamma_0)\right) f(\gamma_0)}{\int_{\gamma=0}^{\infty} \phi\left(\frac{1}{\sigma}\ln(x/\gamma)\right) f(\gamma) d\gamma}. \quad (\text{A.12})$$

Hence, the expected profit gain for the bank that has observed a signal x and knows that all other banks will offer relationship lending if they observe signals less than y is

$$\begin{aligned} \omega_\sigma(x, y) &\equiv \int_{\gamma=0}^{\infty} \omega(\lambda, \gamma) P^\sigma(\gamma|x) d\gamma \\ &= \frac{\int_{\gamma=0}^{\infty} \omega\left(1 - \Phi\left(\frac{1}{\sigma}\ln(y/\gamma)\right), \gamma\right) \phi\left(\frac{1}{\sigma}\ln(x/\gamma)\right) f(\gamma) d\gamma}{\int_{\gamma=0}^{\infty} \phi\left(\frac{1}{\sigma}\ln(x/\gamma)\right) f(\gamma) d\gamma}. \end{aligned} \quad (\text{A.13})$$

Moreover, from the definition (A.13), the expected profit gain $\omega_\sigma(x, y)$ is continuous in x and y , increasing in x and decreasing in y .

First, I want to show there must exist $\underline{\sigma}_1 > 0$ such that the expected profit gain is negative $\omega_\sigma(x, y) < 0$ for all $\sigma \leq \underline{\sigma}_1$, $x \leq \underline{x}_1$ and $y \geq 0$. By the existence of dominance regions (B3), I can choose $\underline{x}_1 < \underline{\gamma}$ and a continuously differentiable function $\bar{\omega} : \mathbf{R}^{++} \rightarrow \mathbf{R}$ with $\bar{\omega}'(\gamma) = 0$ and $\bar{\omega}(\gamma) = -\varepsilon$ for all $\gamma \leq \underline{x}_1$ such that

$$\omega(\lambda, \gamma) \leq \bar{\omega}(\gamma) \leq -\varepsilon,$$

for any proportion $\lambda \in [0, 1]$ and state of liquidity $\gamma \geq 0$. Let $\bar{\omega}_\sigma(x)$ be the upper bound on the expected profit gain $\omega_\sigma(x, y)$ for all $y \geq 0$:

$$\bar{\omega}_\sigma(x) \equiv \int_{\gamma=0}^{\infty} \bar{\omega}(\gamma) P^\sigma(\gamma|x) d\gamma = \frac{\int_{\gamma=0}^{\infty} \bar{\omega}(\gamma) \phi\left(\frac{1}{\sigma}\ln(x/\gamma)\right) f(\gamma) d\gamma}{\int_{\gamma=0}^{\infty} \phi\left(\frac{1}{\sigma}\ln(x/\gamma)\right) f(\gamma) d\gamma}.$$

By changing variable $\ln z = \frac{1}{\sigma}\ln(x/\gamma)$ (or $\gamma = x/z^\sigma$), I have

$$\bar{\omega}_\sigma(x) = \frac{\int_{\ln z=-\infty}^{\infty} \bar{\omega}(x/z^\sigma) \phi(\ln z) f(x/z^\sigma) z^{-\sigma} d\ln z}{\int_{\ln z=-\infty}^{\infty} \phi(\ln z) f(x/z^\sigma) z^{-\sigma} d\ln z}. \quad (\text{A.14})$$

Obviously, from (A.14), $\bar{\omega}_\sigma(x)$ is continuous in σ , and

$$\begin{aligned} \left. \frac{d\bar{\omega}_\sigma(x)}{d\sigma} \right|_{\sigma=0} &= \frac{\int_{\ln z=-\infty}^{\infty} \left[\bar{\omega}'(x/z^\sigma) \frac{x}{z^\sigma} [-\ln z] \phi(\ln z) f(x/z^\sigma) z^{-\sigma} + \bar{\omega}(x/z^\sigma) \frac{d[\phi(\ln z) f(x/z^\sigma) z^{-\sigma}]}{d\sigma} \right] d\ln z}{\int_{\ln z=-\infty}^{\infty} \phi(\ln z) f(x/z^\sigma) z^{-\sigma} d\ln z} \bigg|_{\sigma=0} \\ &\quad - \frac{\left[\int_{\ln z=-\infty}^{\infty} \bar{\omega}(x/z^\sigma) \phi(\ln z) f(x/z^\sigma) z^{-\sigma} d\ln z \right] \left[\int_{\ln z=-\infty}^{\infty} \frac{d[\phi(\ln z) f(x/z^\sigma) z^{-\sigma}]}{d\sigma} d\ln z \right]}{\left[\int_{\ln z=-\infty}^{\infty} \phi(\ln z) f(x/z^\sigma) z^{-\sigma} d\ln z \right]^2} \bigg|_{\sigma=0} \\ &= \left[x \int_{\ln z=-\infty}^{\infty} [-\ln z] \phi(\ln z) d\ln z \right] \bar{\omega}'(x) = 0, \end{aligned}$$

also $\bar{\omega}_0(x) = \bar{\omega}(x)$ so $\bar{\omega}_0(x) = -\varepsilon$ for all $x \leq \underline{x}_1$. The argument shows that there must exist $\underline{\sigma}_1 > 0$ such that $\bar{\omega}_\sigma(x) < 0$ for all $\sigma \leq \underline{\sigma}_1$, $x \leq \underline{x}_1$ and $y \geq 0$. Therefore, the expected profit gain $\omega_\sigma(x, y)$ is negative since $\bar{\omega}_\sigma(x)$ is the upper bound on $\omega_\sigma(x, y)$. By a symmetric argument, there exists $\bar{\sigma}_1 > 0$ such that the expected profit gain is positive $\omega_\sigma(x, y) > 0$ for all $\sigma \leq \bar{\sigma}_1$, $x \geq \bar{x}_1$ and $y \geq 0$.

Second, a strategy survives n rounds of iterated deletion of dominated strategies if and only if

$$s(x) = \begin{cases} 1, & \text{if } x > \bar{x}_n \\ 0, & \text{if } x < \underline{x}_n \end{cases},$$

where $\bar{x}_0 = +\infty$ and $\underline{x}_0 = 0$. In addition, \bar{x}_n and \underline{x}_n are defined by

$$\begin{aligned} \bar{x}_n &= \max \{x : \omega_\sigma(x, \bar{x}_{n-1}) = 0\}, \\ \underline{x}_n &= \min \{x : \omega_\sigma(x, \underline{x}_{n-1}) = 0\}. \end{aligned}$$

If transactional lending (action 1) were to be a best response to a strategy surviving n rounds of iterated deletion of dominated strategies, it must be a best response to the strategy with threshold \underline{x}_n . That is, \underline{x}_{n+1} is defined to be the lowest signal where this occurs. Similarly, if relationship lending (action 0) were to be a best response to a strategy surviving n rounds of iterated deletion of dominated strategies, it must be a best response to the strategy with threshold \bar{x}_n . That is, \bar{x}_{n+1} is defined to be the highest signal where this occurs. By strategic complementarities (B2), \bar{x}_n and \underline{x}_n are decreasing and increasing sequences, respectively. Thus $\bar{x}_n \rightarrow \bar{x}$ and $\underline{x}_n \rightarrow \underline{x}$ as $n \rightarrow \infty$. The continuity of the expected profit gain $\omega_\sigma(x, y)$ and the construction of \bar{x} and \underline{x} imply that $\omega_\sigma(\bar{x}, \bar{x}) = 0$ and $\omega_\sigma(\underline{x}, \underline{x}) = 0$.

Finally, I want to show there is unique $x = x^*$ such that $\omega_\sigma^*(x, x) \equiv \omega_\sigma(x^*, x^*) = 0$. Let $\Psi^\sigma(\lambda|x, y)$ be the probability that the bank, that has observed signal x , assigns to proportion less than or equal to λ of the other banks observing a signal greater than or equal to y . Given the posterior (A.12), I have

$$\begin{aligned} \Psi^\sigma(\lambda|x, y) &= \int_{\gamma=0}^y \frac{\exp(-\sigma\Phi^{-1}(1-\lambda))}{P^\sigma(\gamma|x)} P^\sigma(\gamma|x) d\gamma \\ &= \frac{\int_{\gamma=0}^y \exp(-\sigma\Phi^{-1}(1-\lambda)) \phi\left(\frac{1}{\sigma} \ln(x/\gamma)\right) f(\gamma) d\gamma}{\int_{\gamma=0}^{\infty} \phi\left(\frac{1}{\sigma} \ln(x/\gamma)\right) f(\gamma) d\gamma}. \end{aligned} \tag{A.15}$$

Thus the equation (A.13) can be written as

$$\omega_{\sigma}(x, y) = \int_{\lambda=0}^1 \omega(\lambda, y \exp(-\sigma \Phi^{-1}(1-\lambda))) \Psi^{\sigma}(d\lambda|x, y). \quad (\text{A.16})$$

Again, by changing variable $\ln z = \frac{1}{\sigma} \ln(x/\gamma)$, the equation (A.15) can be written as

$$\Psi^{\sigma}(\lambda|x, y) = \frac{\int_{\ln z = \frac{1}{\sigma} \ln(x/y) + \Phi^{-1}(1-\lambda)}^{\infty} \phi(\ln z) f(x/z^{\sigma}) z^{-\sigma} d \ln z}{\int_{\ln z = -\infty}^{\infty} \phi(\ln z) f(x/z^{\sigma}) z^{-\sigma} d \ln z}.$$

For small σ , the shape of the prior will not matter and the posterior beliefs over the proportion λ will depend only on $\frac{1}{\sigma} \ln(x/y)$. Setting $k = \frac{1}{\sigma} \ln(x/y)$, I have

$$\lim_{\sigma \rightarrow 0} \Psi^{\sigma}(\lambda|x, y) = \int_{\ln z = k + \Phi^{-1}(1-\lambda)}^{\infty} \phi(\ln z) d \ln z = 1 - \Phi(k + \Phi^{-1}(1-\lambda)).$$

Now, if $x = y$, then this becomes an identity function,

$$\lim_{\sigma \rightarrow 0} \Psi^{\sigma}(\lambda|x, x) = \lambda, \quad (\text{A.17})$$

It is the cumulative distribution function of the uniform density.

From equations (A.16) and (A.17), banks take the Laplacian action: Relationship lending (action 0) is the chosen action at x if

$$\lim_{\sigma \rightarrow 0} \int_{\lambda=0}^1 \omega(\lambda, \gamma) \Psi^{\sigma}(d\lambda|x, x) = \int_{\lambda=0}^1 \omega(\lambda, x) d\lambda < 0,$$

and transactional lending (action 1) is the chosen action if the opposite holds. By state monotonicity (B1), the threshold $x = x^*$ is unique and will be solved from equation (1.18). Q.E.D.

APPENDIX B. ADDITIONAL MATERIAL FOR CHAPTER 2

Proofs

Proof of Lemma 2.1: Let $\bar{\eta}_\ell = \sup_{s_{\text{pub}}} \eta(s_{\text{pub}}|\ell)$, and $\underline{\eta}_\ell = \inf_{s_{\text{pub}}} \eta(s_{\text{pub}}|\ell)$. In addition, let $\underline{\nu}_{s_{\text{pub}},\ell} = \inf_{s_{\text{priv}}} \nu(s_{\text{priv}}|s_{\text{pub}},\ell)$, and $\bar{\nu}_{s_{\text{pub}},\ell} = \sup_{s_{\text{priv}}} \nu(s_{\text{priv}}|s_{\text{pub}},\ell)$. Integrating by parts,

$$\begin{aligned} \eta(s_{\text{pub}}|\ell) &= \int_{\theta} \theta dF(\theta|s_{\text{pub}},\ell) = \\ - \int_{\theta} \theta d(1 - F(\theta|s_{\text{pub}},\ell)) &= \int_{\theta} (1 - F(\theta|s_{\text{pub}},\ell)) d\theta. \end{aligned}$$

Hence, $\frac{\partial}{\partial s_{\text{pub}}} \eta(s_{\text{pub}}|\ell)$ equals $-\int_{\theta} \frac{\partial F(\theta|s_{\text{pub}},\ell)}{\partial s_{\text{pub}}} d\theta$ which, by Public Signal Monotonicity, exists and lies in $[\int_{\theta} \underline{\lambda}(\theta) d\theta, \int_{\theta} \bar{\lambda}(\theta) d\theta]$. In addition,

$$\begin{aligned} E(\theta|s_{\text{priv}},s_{\text{pub}},\ell) &= \int_{\theta} \theta dF(\theta|s_{\text{priv}},s_{\text{pub}},\ell) = \\ - \int_{\theta} \theta d(1 - F(\theta|s_{\text{priv}},s_{\text{pub}},\ell)) &= \int_{\theta} (1 - F(\theta|s_{\text{priv}},s_{\text{pub}},\ell)) d\theta. \end{aligned}$$

Thus, $\frac{\partial}{\partial s_{\text{priv}}} E(\theta|s_{\text{priv}},s_{\text{pub}},\ell) = -\int_{\theta} \frac{\partial F(\theta|s_{\text{priv}},s_{\text{pub}},\ell)}{\partial s_{\text{priv}}} d\theta$ which, by Private Signal Monotonicity, exists and lies in $[\int_{\theta} \underline{\mu}(\theta) d\theta, \int_{\theta} \bar{\mu}(\theta) d\theta]$. Since $\eta \in (0, 1)$, the slope of $\nu(s_{\text{priv}}|s_{\text{pub}},\ell)$ lies in \mathfrak{R}_{++} . Q.E.D.

Proof of Lemma 2.2: The proof of Lemma 2.1 implies that (a) $\eta(s_{\text{pub}}|\ell)$ has a differentiable inverse function $s_{\text{pub}}(\eta|\ell)$ of η , which is a bijection from $(\underline{\eta}_\ell, \bar{\eta}_\ell) \subset [0, 1]$ to $(0, 1)$ whose slope lies in \mathfrak{R}_{++} and (b) $\nu(s_{\text{priv}}|s_{\text{pub}},\ell)$ has a differentiable inverse function $s_{\text{priv}}(\nu|s_{\text{pub}},\ell)$ of ν , which is a bijection from $(\underline{\nu}_{s_{\text{pub}},\ell}, \bar{\nu}_{s_{\text{pub}},\ell}) \subset [0, \eta(s_{\text{pub}}|\ell)^{-1}]$ to $(0, 1)$ whose slope lies in \mathfrak{R}_{++} . We now extend the function $s_{\text{pub}}(\eta|\ell)$ to all pairs (η, ℓ) in $(0, 1) \times [0, 1]$ by defining it as one if $\eta > \sup_{s_{\text{pub}}} \eta(s_{\text{pub}}|\ell)$ and zero if $\eta < \inf_{s_{\text{pub}}} \eta(s_{\text{pub}}|\ell)$. By Lemma 2.1,

$$G(\eta_0, \ell_0) = \Pr(s_{\text{pub}} < s_{\text{pub}}(\eta_0|\ell) \text{ and } \ell \leq \ell_0) = \int_{\ell=0}^{\ell_0} \int_{s_{\text{pub}}=-\infty}^{s_{\text{pub}}(\eta_0|\ell)} f(s_{\text{pub}}, \ell) ds_{\text{pub}} d\ell.$$

Hence, $g(\eta_0, \ell_0) = \frac{\partial^2 G(\eta_0, \ell_0)}{\partial \eta_0 \partial \ell_0} = f(s_{\text{pub}}(\eta_0 | \ell_0), \ell_0) \frac{\partial s_{\text{pub}}(\eta_0 | \ell_0)}{\partial \eta_0}$ where $f(s_{\text{pub}}(\eta_0 | \ell_0), \ell_0)$ denotes the marginal density $f(s_{\text{pub}}, \ell)$ evaluated at $(s_{\text{pub}}(\eta_0 | \ell_0), \ell_0)$. Since (η_0, ℓ_0) is feasible, we have the following $\inf_{s_{\text{pub}}} \eta(s_{\text{pub}} | \ell_0) < \eta_0 < \sup_{s_{\text{pub}}} \eta(s_{\text{pub}} | \ell_0)$, whence $s_{\text{pub}}(\eta_0 | \ell_0)$ lies in $(0, 1)$ by Lemma 2.1. Thus, $f(s_{\text{pub}}(\eta_0 | \ell_0), \ell_0) \in \mathfrak{R}_{++}$ by assumption. Lemma 2.1 implies further that $\frac{\partial s_{\text{pub}}(\eta_0 | \ell_0)}{\partial \eta_0} \in \mathfrak{R}_{++}$. Thus, $g(\eta_0, \ell_0) \in \mathfrak{R}_{++}$ as claimed. Q.E.D.

Proof of Lemma 2.3: First,

$$\begin{aligned} H(\nu_0 | \eta_0, \ell_0) &= \Pr(\nu(s_{\text{priv}} | s_{\text{pub}}, \ell) \leq \nu_0 | s_{\text{pub}} = s_{\text{pub}}(\eta_0 | \ell_0), \ell = \ell_0) \\ &= \Pr(s_{\text{priv}} \leq s_{\text{priv}}(\nu_0 | s_{\text{pub}}, \ell) | s_{\text{pub}} = s_{\text{pub}}(\eta_0 | \ell_0), \ell = \ell_0) \\ &= F(s_{\text{priv}}(\nu_0 | s_{\text{pub}}, \ell) | s_{\text{pub}}(\eta_0 | \ell_0), \ell_0). \end{aligned}$$

Hence,

$$\begin{aligned} H(\nu | \eta, \ell) &= F(s_{\text{priv}}(\nu | s_{\text{pub}}, \ell) | s_{\text{pub}}(\eta | \ell), \ell), \\ H'(\nu | \eta, \ell) &= F'(s_{\text{priv}}(\nu | s_{\text{pub}}, \ell) | s_{\text{pub}}(\eta | \ell), \ell) s'_{\text{priv}}(\nu | s_{\text{pub}}, \ell), \text{ and} \\ H''(\nu | \eta, \ell) &= F''(s_{\text{priv}}(\nu | s_{\text{pub}}, \ell) | s_{\text{pub}}(\eta | \ell), \ell) [s'_{\text{priv}}(\nu | s_{\text{pub}}, \ell)]^2 \\ &\quad + F'(s_{\text{priv}}(\nu | s_{\text{pub}}, \ell) | s_{\text{pub}}(\eta | \ell), \ell) s''_{\text{priv}}(\nu | s_{\text{pub}}, \ell). \end{aligned}$$

Differentiating the identity $\nu(s_{\text{priv}}(\nu | s_{\text{pub}}, \ell) | s_{\text{pub}}, \ell) = \nu$ with respect to ν ,

$$\begin{aligned} 1 &= \nu'(s_{\text{priv}}(\nu | s_{\text{pub}}, \ell) | s_{\text{pub}}, \ell) s'_{\text{priv}}(\nu | s_{\text{pub}}, \ell) \text{ and} \\ 0 &= \nu'(s_{\text{priv}}(\nu | s_{\text{pub}}, \ell) | s_{\text{pub}}, \ell) s''_{\text{priv}}(\nu | s_{\text{pub}}, \ell) \\ &\quad + \nu''(s_{\text{priv}}(\nu | s_{\text{pub}}, \ell) | s_{\text{pub}}, \ell) [s'_{\text{priv}}(\nu | s_{\text{pub}}, \ell)]^2 \end{aligned}$$

so that

$$\begin{aligned} s'_{\text{priv}}(\nu | s_{\text{pub}}, \ell) &= [\nu'(s_{\text{priv}}(\nu | s_{\text{pub}}, \ell) | s_{\text{pub}}, \ell)]^{-1} = [\nu'(s_{\text{priv}} | s_{\text{pub}}, \ell)]^{-1} \text{ and} \\ s''_{\text{priv}}(\nu | s_{\text{pub}}, \ell) &= -\frac{\nu''(s_{\text{priv}} | s_{\text{pub}}, \ell)}{[\nu'(s_{\text{priv}} | s_{\text{pub}}, \ell)]^3}. \end{aligned}$$

Accordingly,

$$\frac{H''(\nu | \eta, \ell) \nu}{H'(\nu | \eta, \ell)} = \frac{\left(F'' [s'_{\text{priv}}]^2 + F' s''_{\text{priv}} \right) \nu}{F' s'_{\text{priv}}} = \frac{F''}{F' \nu'} - \frac{\nu'' \nu}{[\nu']^2}.$$

Q.E.D.

Proof of Lemma 2.4: Consider, for instance, bank a . If it competes for a borrower in region B , it must make the first offer. It does not know the borrower's private type ν . Since bank b knows ν , bank b can tell which of bank a 's offers are profitable for bank a and which are not. However, in the absence of securitization the two banks have common values: the value of lending to a borrower is simply her discounted expected repayment less the common cost of capital. Thus, bank b will slightly underbid bank a 's profitable offers and refrain from bidding on the unprofitable ones. As a result, bank a will succeed in lending only to unprofitable borrowers. Knowing this, bank a will not make offers to any agents who reside in region B in period 1. But given this, in period 2 bank b can charge the maximum possible interest rate of ρ and any of these agents will agree. It will do so if the resulting discounted expected repayment, $\delta\rho\eta\nu E(L_\ell^R)$, exceeds its unitary cost of capital. Q.E.D.

Proof of Lemma 2.6: Equilibrium requires that bank a does not want to change its strategy taking bank b 's strategy q_b as given. Define

$$P_a(q, \sigma) = \int_{u^b \in \mathbb{R}_+^N} p_a(q, q_b(u^b), \sigma) d\Psi(u^b|\sigma)$$

and

$$\Phi_a(u^a, \sigma) = \int_{u^b \in \mathbb{R}_+^N} \phi_a(u^a, u^b, \sigma) d\Psi(u^b|\sigma).$$

Integrating conditions 1 and 3 over bank b 's possible signal vectors u^b , we find that, almost surely, bank a 's optimal quantity q maximizes $q[P_a(q, \sigma) - \delta\Phi_a(u^a, \sigma)]$ and $P_a(q_a(u^a, \sigma), \sigma)$ equals $\Phi_a(u^a, \sigma)$. These are the conditions for a separating equilibrium of the single-sender game analyzed by DeMarzo and Duffie [16, p. 77]. By their Proposition 2 [16, p. 78], bank a sells a quantity $q_a(u^a, \sigma) = \left[\hat{\phi}_a(0, \sigma) / \hat{\phi}_a(u^a, \sigma) \right]^{\frac{1}{1-\delta}}$ and the expected market price when bank a sells a quantity q is $P_a(q, \sigma) = \hat{\phi}_a(0, \sigma) / q^{1-\delta}$. The quantity and expected price of bank a 's security thus does not depend on bank b 's strategy since $\hat{\phi}_a(u^a, \sigma)$ does not. Hence, DeMarzo and Duffie's equation (4) [16, p. 79] implies that in any separating equilibrium, bank a 's securitization profits conditional on the signals u^a and σ are given by $\pi(u^a, \sigma) = (1 - \delta) \hat{\phi}_a(0, \sigma)^{\frac{1}{1-\delta}} \hat{\phi}_a(u^a, \sigma)^{-\frac{\delta}{1-\delta}}$ as claimed.

It remains to show that bank a 's optimal security is debt. Following DeMarzo and Duffie [16, pp. 88-89], let $\varphi_a(\cdot)$ be any monotone security. Since Y_a is nondecreasing in each factor ζ_k^A , for each public signal σ the lowest possible realization of $E(\varphi_a(Y_a)|u^a, \sigma)$ is $E(\varphi_a(Y_a)|u^a = 0; \sigma)$. Now consider a standard debt security $\min\{m_a, Y_a\}$. By the dominated convergence theorem, $E(\min\{m_a, Y_a\}|u^a = 0; \sigma)$ is continuous in m_a , so we may choose m_a so that the following holds $E(\min\{m_a, Y_a\}|u^a = 0; \sigma) = E(\varphi_a(Y_a)|u^a = 0; \sigma)$. Let $d(Y_a) = \varphi_a(Y_a) - \min\{m_a, Y_a\}$ and $\psi(u^a, \sigma) = E(d(Y_a)|u^a, \sigma)$; by construction, $\psi(0, \sigma) = 0$. Because $\varphi_a(Y_a) \leq Y_a$, for $Y_a \leq m_a$ we have $d(Y_a) = \varphi_a(Y_a) - Y_a \leq 0$. Moreover, for $Y_a \geq m_a$, $d(Y_a) = \varphi_a(Y_a) - m_a$, which is nondecreasing in Y_a . Hence, there is a $y^* \in [m_a, \infty) \cup \{\infty\}$ such that $d(Y_a) > 0$ if and only if $Y_a > y^*$. Moreover, since the measure of agents is 2 and each is willing to pay at most ρ , Y_a is bounded by 2ρ . Let $\mu(y|u^a, \sigma)$ be the conditional density of Y_a at the realization y given the signals u^a and σ . Since the conditional (on u^a and σ) distribution of ζ^a is mutually absolutely continuous with respect to u^a , the conditional density has a well defined Radon-Nikodym derivative $\frac{\mu(y|u^a, \sigma)}{\mu(y|0, \sigma)}$ for each public signal σ . As noted by DeMarzo and Duffie [16, p. 88, n. 30], the measure μ can be chosen so that the Radon-Nikodym derivative $\frac{\mu(y|u^a, \sigma)}{\mu(y|0, \sigma)}$ is nondecreasing in y . Thus, for any signal vector u^a ,

$$\begin{aligned} \psi(u^a, \sigma) &= E(d(Y_a)|u^a, \sigma) = \int_{y=0}^{2\rho} d(y) \mu(y|u^a, \sigma) dy = \int_{y=0}^{2\rho} d(y) \frac{\mu(y|u^a, \sigma)}{\mu(y|0, \sigma)} \mu(y|0, \sigma) dy \\ &\geq \int_{y=0}^{2\rho} d(y) \frac{\mu(y^*|u^a, \sigma)}{\mu(y^*|0, \sigma)} \mu(y|0, \sigma) dy = \frac{\mu(y^*|u^a, \sigma)}{\mu(y^*|0, \sigma)} \int_{y=0}^{2\rho} d(y) \mu(y|0, \sigma) dy = 0 \end{aligned}$$

Thus, $E[\varphi_a(Y_a)|u^a, \sigma] = E[\min\{m_a, Y_a\}|u^a, \sigma] + \psi(u^a, \sigma) \geq E[\min\{m_a, Y_a\}|u^a, \sigma]$. Hence, by switching from the security $\varphi_a(Y_a)$ to the security $\min\{m_a, Y_a\}$, the bank weakly lowers $\hat{\phi}_a(u^a, \sigma)$ (the expected payout of the security conditional on u^a and σ) while not changing $\hat{\phi}_a(0, \sigma)$, thus weakly raising conditional profits $\pi(u^a, \sigma)$ and thus unconditional profits $E[\pi(u^a, \sigma)]$. This shows that the optimal security is debt. Q.E.D.

For the remainder, we need additional notation. Let $z_{\eta\ell}^B = r_{\eta\ell}^B p_{\eta\ell}^B$ denote the product of the interest rate charged to region B borrowers with credit score η in location ℓ and the proportion of these loans that are securitized. This quantity, which must lie between zero and $r_{\eta\ell}^B$, can be interpreted as the amount of loans that bank a securitizes, expressed in units of the face

value $r_{\eta\ell}^B$ of these loans. Given $r_{\eta\ell}^B$, choosing $p_{\eta\ell}^B$ is clearly equivalent to choosing $z_{\eta\ell}^B$. With this change of variables,

$$Y_a^B(\zeta^B) = \int_{\ell=0}^1 \int_{\eta=0}^1 x_{\eta\ell}^B z_{\eta\ell}^B \left[\eta L_{\eta\ell}^B \int_{\nu=0}^{\underline{\nu}_{\eta\ell}^B} \nu dH(\nu|\eta, \ell) \right] dG(\eta, \ell). \quad (\text{B.1})$$

Bank a 's Lagrangean equals its expected payoff Π_a plus constraint terms, which we write in a manner analogous to the integrals that appear in Π_a :

$$\begin{aligned} \mathcal{L} = & \Pi_a + \int_{\ell=0}^1 \int_{\eta=0}^1 (a_{\eta\ell} z_{\eta\ell}^B + b_{\eta\ell} (r_{\eta\ell}^B - z_{\eta\ell}^B)) \eta (1 - H(\underline{\nu}_{\eta\ell}^B|\eta, \ell)) dG(\eta, \ell) \\ & + \int_{\ell=0}^1 \int_{\eta=0}^1 \left[c_{\eta\ell} r_{\eta\ell}^A \eta \int_{\nu=\underline{\nu}_{\eta\ell}^A}^{\bar{\nu}_{\eta\ell}} \nu dH(\nu|\eta, \ell) + d_{\eta\ell} r_{\eta\ell}^A \eta \int_{\nu=\underline{\nu}_{\eta\ell}^A}^{\bar{\nu}_{\eta\ell}^A} \nu dH(\nu|\eta, \ell) \right] dG(\eta, \ell) \end{aligned}$$

where $a_{\eta\ell}$, $b_{\eta\ell}$, $c_{\eta\ell}$, and $d_{\eta\ell}$ are Lagrange multipliers for the constraints $z_{\eta\ell}^B \geq 0$, $z_{\eta\ell}^B \leq r_{\eta\ell}^B$, $\bar{\nu}_{\eta\ell}^A \leq \bar{\nu}_{\eta\ell}$, and $\underline{\nu}_{\eta\ell}^A \leq \bar{\nu}_{\eta\ell}^A$, respectively. For technical reasons, we omit the constraint $r_{\eta\ell}^B \leq \rho$ and verify later that it holds. Bank b 's Lagrangean, which is analogous, is omitted.

Proof of Lemma 2.7: W.l.o.g. let $i = a$ and $j = b$. Since u^A and ζ^B are independent conditional on σ ,

$$E[1(m_a > Y_a) \zeta_k^B | u^a, \sigma] = \int_{\zeta^B} \zeta_k^B \left[\int_{\zeta^A} 1(m_a > Y_a^B(\zeta^B) + Y_a^A(\zeta^A)) d\Gamma(\zeta^A | u^a, \sigma) \right] d\Gamma(\zeta^B | \sigma),$$

where $\Gamma(\zeta^B | \sigma)$ is the distribution function of ζ^B conditional on σ . By stochastic dominance, the interior integral is nonincreasing in u^a , so the double integral is as well. But stochastic dominance also implies $\hat{\phi}_a(0, \sigma) \leq \hat{\phi}_a(u^a, \sigma)$. Hence $\Lambda_{ak}^{B0}(\sigma) \geq \Lambda_{ak}^B(\sigma)$ for all public signals σ , which proves that $\Omega_{al}^B(\sigma) > 0$ since $\delta < 1$. Hence, $\Omega_{al}^B = E(\Omega_{al}^B(\sigma)) > 0$. Q.E.D.

Proof of Theorem 2.8: the proof consists of the following claims.

Lemma B.1 *If bank a competes for region B borrowers with credit score η in location ℓ , it bids a strictly positive interest rate $r_{\eta\ell}^B$.*

Lemma B.2 *If bank a competes for customers with credit score η in location ℓ in region B , then it includes all of them in its security: if $x_{\eta\ell}^B = 1$, then $p_{\eta\ell}^B = 1$.*

Lemma B.3 *Consider the group of agents with credit score η living in location ℓ in region B . Given the interest rate $r_{\eta\ell}^B$ offered by bank a , bank b responds as follows.*

1. It lends to all agents whose private type ν exceeds

$$\underline{\nu}_{\eta\ell}^B = \min \{ \bar{\nu}_{\eta\ell}, \bar{\nu}_{\eta\ell} r_{\eta\ell}^{B*} / r_{\eta\ell}^B \}. \quad (\text{B.2})$$

In particular, it strictly prefers (not) to lend when a borrower's private type ν exceeds (respectively, is less than) $\bar{\nu}_{\eta\ell} r_{\eta\ell}^{B*} / r_{\eta\ell}^B$, and is indifferent when ν equals this expression.

2. If bank b lends to some region B borrowers in this group (i.e., if $\underline{\nu}_{\eta\ell}^B < \bar{\nu}_{\eta\ell}$), then it securitizes all of these borrowers if $\Omega_{b\ell}^B > 0$ and none of them if $\Omega_{b\ell}^B < 0$.

Lemma B.4 *If bank a competes for region B borrowers with credit score η in location ℓ (if $x_{\eta\ell}^B = 1$), it offers the interest rate $r_{\eta\ell}^B = \min \{ \rho, r_{\eta\ell}^{B*} \}$ and lends to all borrowers in this group. If $r_{\eta\ell}^{B*} \leq \rho$, then bank b is just willing not to bid for the best borrower in this group: the borrower whose private type ν is $\bar{\nu}_{\eta\ell}$. If $r_{\eta\ell}^{B*} > \rho$, bank b strictly prefers not to bid for any borrowers in the group.*

Lemma B.5 *Bank a competes for region B borrowers with credit score η in location ℓ (i.e., it sets $x_{\eta\ell}^B = 1$) if and only if*

$$r_{\eta\ell}^B [\delta E(L_{\ell}^B) + \Omega_{a\ell}^B] \eta > 1. \quad (\text{B.3})$$

This concludes the proof of Theorem 2.8. Q.E.D.

Proof of Lemma B.1: Suppose otherwise: $r_{\eta\ell}^B = 0$. Since $r_{\eta\ell}^B = 0$, $\partial X_a^B / \partial x_{\eta\ell}^B = 0$ and $\partial Y_a^B / \partial x_{\eta\ell}^B = 0$ (since $z_{\eta\ell}^B \leq r_{\eta\ell}^B$). Hence,

$$\frac{\partial \mathcal{L}}{\partial x_{\eta\ell}^B} = \delta E \left(\frac{\partial X_a^B}{\partial x_{\eta\ell}^B} \right) - \frac{\partial C_a^B}{\partial x_{\eta\ell}^B} + (1 - \delta) E \left(\frac{\partial}{\partial x_{\eta\ell}^B} \frac{\hat{\phi}_a(0, \sigma)^{\frac{1}{1-\delta}}}{\hat{\phi}_a(u^a, \sigma)^{\frac{\delta}{1-\delta}}} \right) = -\frac{\partial C_a^B}{\partial x_{\eta\ell}^B} < 0.$$

Thus, $x_{\eta\ell}^B = 0$. Q.E.D.

Proof of Lemma B.2: The first order condition for $z_{\eta\ell}^B$ is

$$0 = E \left(\frac{\hat{\phi}_a(0, \sigma)^{\frac{1}{1-\delta}}}{\hat{\phi}_a(u^a, \sigma)^{\frac{\delta}{1-\delta}}} \left(\frac{E \left(1(m_a > Y_a) \frac{\partial Y_a}{\partial z_{\eta\ell}^B} \Big| u^a = 0; \sigma \right)}{\hat{\phi}_a(0, \sigma)} - \delta \frac{E \left(1(m_a > Y_a) \frac{\partial Y_a}{\partial z_{\eta\ell}^B} \Big| u^a, \sigma \right)}{\hat{\phi}_a(u^a, \sigma)} \right) \right) \\ + (a_{\eta\ell} - b_{\eta\ell}) \eta g(\eta, \ell) \int_{\nu=0}^{\underline{\nu}_{\eta\ell}^B} \nu dH(\nu | \eta, \ell)$$

However,

$$\frac{1}{\eta g(\eta, \ell) \int_{\nu=0}^{\nu_{\eta\ell}^B} \nu dH(\nu|\eta, \ell)} \frac{\partial Y_a}{\partial z_{\eta\ell}^B} = x_{\eta\ell}^B L_\ell^B = x_{\eta\ell}^B \sum_{k=1}^K \alpha_{k\ell} \zeta_k^B$$

Hence,

$$b_{\eta\ell} - a_{\eta\ell} = \begin{cases} 0 & \text{if } x_{\eta\ell}^B = 0 \\ \Omega_{a\ell}^B & \text{if } x_{\eta\ell}^B = 1 \end{cases} \quad (\text{B.4})$$

By Lemma 2.7, $\Omega_{a\ell}^B > 0$, whence $b_{\eta\ell} - a_{\eta\ell} > 0$ if $x_{\eta\ell}^B = 1$. But $a_{\eta\ell}$ and $b_{\eta\ell}$ are the Lagrange multipliers for the constraints $z_{\eta\ell}^B \geq 0$ and $z_{\eta\ell}^B \leq r_{\eta\ell}^B$, respectively. Thus, by Lemma B.1, either $a_{\eta\ell}$ or $b_{\eta\ell}$ must be zero. Together with (B.4), this implies that $b_{\eta\ell} = \Omega_{a\ell}^B > 0 = a_{\eta\ell}$, so $0 < z_{\eta\ell}^B = r_{\eta\ell}^B$. Q.E.D.

Proof of Lemma B.3: The derivatives of the bank's profits Π_b and the Lagrangean \mathcal{L} with respect to $\bar{\nu}_{\eta\ell}^B$ are

$$\frac{\partial \Pi_b}{\partial \bar{\nu}_{\eta\ell}^B} = r_{\eta\ell}^B \eta \bar{\nu}_{\eta\ell}^B H'(\bar{\nu}_{\eta\ell}^B | \eta, \ell) g(\eta, \ell) \Omega_{b\ell}^B \quad (\text{B.5})$$

and

$$\frac{\partial \mathcal{L}}{\partial \bar{\nu}_{\eta\ell}^B} = r_{\eta\ell}^B \eta \bar{\nu}_{\eta\ell}^B H'(\bar{\nu}_{\eta\ell}^B | \eta, \ell) g(\eta, \ell) [\Omega_{b\ell}^B + d_{\eta\ell} - c_{\eta\ell}]. \quad (\text{B.6})$$

Since this must equal zero, it follows that

$$c_{\eta\ell} - d_{\eta\ell} = \Omega_{b\ell}^B. \quad (\text{B.7})$$

The derivatives of the bank's profits Π_b and the Lagrangean with respect to $\underline{\nu}_{\eta\ell}^B$ are

$$\frac{\partial \Pi_b}{\partial \underline{\nu}_{\eta\ell}^B} = (1 - \delta E(r_{\eta\ell}^B \eta \underline{\nu}_{\eta\ell}^B L_\ell^B) - r_{\eta\ell}^B \eta \underline{\nu}_{\eta\ell}^B \Omega_{b\ell}^B) H'(\underline{\nu}_{\eta\ell}^B | \eta, \ell) g(\eta, \ell) \quad (\text{B.8})$$

and

$$\frac{\partial \mathcal{L}}{\partial \underline{\nu}_{\eta\ell}^B} = \frac{\partial \Pi_b}{\partial \underline{\nu}_{\eta\ell}^B} - d_{\eta\ell} r_{\eta\ell}^B \eta \underline{\nu}_{\eta\ell}^B H'(\underline{\nu}_{\eta\ell}^B | \eta, \ell) g(\eta, \ell). \quad (\text{B.9})$$

First, suppose $\underline{\nu}_{\eta\ell}^B = \bar{\nu}_{\eta\ell}$. Then $\bar{\nu}_{\eta\ell}^B = \bar{\nu}_{\eta\ell}$ as well, so $\frac{\partial \Pi_b}{\partial \underline{\nu}_{\eta\ell}^B} \geq 0$ and $\frac{\partial \Pi_b}{\partial \underline{\nu}_{\eta\ell}^B} + \frac{\partial \Pi_b}{\partial \bar{\nu}_{\eta\ell}^B} \geq 0$. (The latter condition means that it is not optimal for the bank to lower both $\underline{\nu}_{\eta\ell}^B$ and $\bar{\nu}_{\eta\ell}^B$ while keeping them equal.) These two inequalities hold if and only if $1 - \delta E(r_{\eta\ell}^B \bar{\nu}_{\eta\ell} \eta L_\ell^B) - r_{\eta\ell}^B \bar{\nu}_{\eta\ell} \eta (\Omega_{b\ell}^B)^+ \geq 0$ which holds if and only if $r_{\eta\ell}^{B*} \geq r_{\eta\ell}^B$ by (2.10). This confirms that (B.2) holds when $\underline{\nu}_{\eta\ell}^B = \bar{\nu}_{\eta\ell}$.

Now suppose $\underline{\nu}_{\eta\ell}^B < \bar{\nu}_{\eta\ell}$. Recall that $c_{\eta\ell}$ and $d_{\eta\ell}$ are the Lagrange multipliers for the constraints $\bar{\nu}_{\eta\ell}^B \leq \bar{\nu}_{\eta\ell}$ and $\underline{\nu}_{\eta\ell}^B \leq \bar{\nu}_{\eta\ell}$, respectively. Only one of these can bind since $\underline{\nu}_{\eta\ell}^B < \bar{\nu}_{\eta\ell}$.

Hence, either $c_{\eta\ell}$ or $d_{\eta\ell}$ is zero. Thus, by (B.7), $c_{\eta\ell} = (\Omega_{b\ell}^B)^+$ while $d_{\eta\ell} = (-\Omega_{b\ell}^B)^+$. Hence, bank b securitizes all of its borrowers in the group ($c_{\eta\ell} > 0$) if $\Omega_{b\ell}^B > 0$, and none of them ($d_{\eta\ell} > 0$) if $\Omega_{b\ell}^B < 0$, as claimed. Moreover, by (B.7), (B.8), and (B.9), $0 = 1 - \delta E \left(r_{\eta\ell}^B \eta \nu_{\eta\ell}^B L_{\ell}^B \right) - r_{\eta\ell}^B \eta \nu_{\eta\ell}^B (\Omega_{b\ell}^B)^+$, which is equivalent to $\nu_{\eta\ell}^B = \bar{\nu}_{\eta\ell} r_{\eta\ell}^{B*} / r_{\eta\ell}^B$. This shows that (B.2) holds when $\nu_{\eta\ell}^B < \bar{\nu}_{\eta\ell}$ as well. Hence, (B.2) always holds. Finally, since only bank b knows ν , it strictly prefers (not) to lend to borrowers whose types ν exceed (respectively, are less than) $\bar{\nu}_{\eta\ell} r_{\eta\ell}^{B*} / r_{\eta\ell}^B$. Q.E.D.

Proof of Lemma B.4: The Lagrangean is not differentiable at the optimal interest rate $r_{\eta\ell}^B$. Hence, to find this optimal rate, we must consider the part of the Lagrangean in which $r_{\eta\ell}^B$ or $\nu_{\eta\ell}^B$ (which depends on $r_{\eta\ell}^B$) appears. It is

$$\begin{aligned} & \delta E (X_a^B) - C_a^B + (1 - \delta) E \left(\frac{\hat{\phi}_a(0, \sigma)^{\frac{1}{1-\delta}}}{\hat{\phi}_a(u^a, \sigma)^{\frac{\delta}{1-\delta}}} \right) \\ & + \int_{\ell=0}^1 \int_{\eta=0}^1 \int_{\nu=0}^{\nu_{\eta\ell}^B} (a_{\eta\ell} z_{\eta\ell}^B + b_{\eta\ell} (r_{\eta\ell}^B - z_{\eta\ell}^B)) \eta \nu dH(\nu|\eta, \ell) dG(\eta, \ell). \end{aligned}$$

In addition, since the choice of $r_{\eta\ell}^B$ does not affect terms that involve credit scores $\eta' \neq \eta$ and locations $\ell' \neq \ell$, the optimal $r_{\eta\ell}^B$ is chosen to maximize

$$\begin{aligned} & \delta E (L_{\ell}^B) \eta \int_{\nu=0}^{\nu_{\eta\ell}^B} r_{\eta\ell}^B \nu dH(\nu|\eta, \ell) - \int_{\nu=0}^{\nu_{\eta\ell}^B} dH(\nu|\eta, \ell) \\ & + (1 - \delta) E \left(\frac{\hat{\phi}_a(0, \sigma)^{\frac{1}{1-\delta}}}{\hat{\phi}_a(u^a, \sigma)^{\frac{\delta}{1-\delta}}} \right) + \int_{\nu=0}^{\nu_{\eta\ell}^B} (a_{\eta\ell} z_{\eta\ell}^B + b_{\eta\ell} (r_{\eta\ell}^B - z_{\eta\ell}^B)) \eta \nu dH(\nu|\eta, \ell). \end{aligned}$$

Now, since $\frac{\partial Y_{\eta\ell}^B}{\partial r_{\eta\ell}^B} = \frac{\partial}{\partial r_{\eta\ell}^B} \int_{\nu=0}^{\nu_{\eta\ell}^B} x_{\eta\ell}^B z_{\eta\ell}^B \eta \nu dH(\nu|\eta, \ell) g(\eta, \ell) L_{\ell}^B$,

$$\frac{\partial}{\partial r_{\eta\ell}^B} \left((1 - \delta) E \left(\frac{\hat{\phi}_a(0, \sigma)^{\frac{1}{1-\delta}}}{\hat{\phi}_a(u^a, \sigma)^{\frac{\delta}{1-\delta}}} \right) \right) = \frac{\partial}{\partial r_{\eta\ell}^B} \left(\int_{\nu=0}^{\nu_{\eta\ell}^B} z_{\eta\ell}^B \eta \nu dH(\nu|\eta, \ell) \right) g(\eta, \ell) \Omega_{a\ell}^B.$$

By Lemma B.2, $a_{\eta\ell} = 0$ and $b_{\eta\ell} = \Omega_{a\ell}^B > 0$. Hence, $r_{\eta\ell}^B$ is chosen to maximize

$$c^{-1} \int_{\nu=0}^{\nu_{\eta\ell}^B} (r_{\eta\ell}^B \nu - c) dH(\nu|\eta, \ell) \stackrel{d}{=} c^{-1} I(r_{\eta\ell}^B) \quad (\text{B.10})$$

where $c^{-1} = \eta [\delta E (L_{\ell}^B) + \Omega_{a\ell}^B]$ is independent of $r_{\eta\ell}^B$. If $r_{\eta\ell}^B < r_{\eta\ell}^{B*}$, by (B.2), $\nu_{\eta\ell}^B$ equals $\bar{\nu}_{\eta\ell}$ so small changes in $r_{\eta\ell}^B$ do not affect it. Hence, (B.10) is strictly increasing in $r_{\eta\ell}^B$, so the optimal $r_{\eta\ell}^B$ is at least $\min \left\{ \rho, r_{\eta\ell}^{B*} \right\}$.

If $r_{\eta\ell}^{B*} \geq \rho$, we are done. If $r_{\eta\ell}^{B*} < \rho$, it suffices to show that the optimal $r_{\eta\ell}^B$ is no greater than $r_{\eta\ell}^{B*}$. Let us write $r = r_{\eta\ell}^B$, $r^* = r_{\eta\ell}^{B*}$, and $H(\nu) = H(\nu|\eta, \ell)$ for brevity, and let $S \subset [0, \bar{\nu}_{\eta\ell}]$ be the support of ν for the given values of η and ℓ . Consider any $r > r^*$. We will show that if $I(r) > 0$, then $I'(r) < 0$. By (B.2), $\underline{\nu}_{\eta\ell}^B = \bar{\nu}_{\eta\ell} r^*/r < \bar{\nu}_{\eta\ell}$, so

$$I(r) = \int_{\nu=0}^{\bar{\nu}_{\eta\ell} r^*/r} (r\nu - c) dH(\nu) = \int_{\nu \in [0, \bar{\nu}_{\eta\ell} r^*/r] \cap S} (r\nu - c) dH(\nu) \quad (\text{B.11})$$

With the change of variables $x = r\nu$, $I(r) = \frac{1}{r} \int_{x \in [0, \bar{\nu}_{\eta\ell} r^*] \cap S'} (x - c) H'\left(\frac{x}{r}\right) dx$ where $S' = \{x \in [0, r] : x/r \in S\}$ is the support of x . Thus,

$$I'(r) = -\frac{1}{r^2} \left(\int_{x \in [0, \bar{\nu}_{\eta\ell} r^*] \cap S'} (x - c) \left[\frac{H'\left(\frac{x}{r}\right) + H''\left(\frac{x}{r}\right) \frac{x}{r}}{H'\left(\frac{x}{r}\right)} \right] H'\left(\frac{x}{r}\right) dx \right).$$

Changing variables back,

$$I'(r) = -\frac{1}{r} \left(\int_{\nu \in [0, \bar{\nu}_{\eta\ell} r^*/r] \cap S} (r\nu - c) \left[\frac{H'(\nu) + H''(\nu)\nu}{H'(\nu)} \right] dH(\nu) \right). \quad (\text{B.12})$$

For any functions $\varphi_0(\nu)$ and $\varphi_1(\nu)$, let $E^*(\varphi_0)$ and $Cov^*(\varphi_0, \varphi_1)$ denote the expectation of φ_0 and covariance of φ_0 and φ_1 , both conditional on $\nu \in [0, \bar{\nu}_{\eta\ell} r^*/r] \cap S$. Then $I'(r) = -\frac{1}{r} E^*(xy) H(\bar{\nu}_{\eta\ell} r^*/r)$, where $x(\nu) = r\nu - c$ and $y(\nu) = \frac{H'(\nu) + H''(\nu)\nu}{H'(\nu)}$. By definition of covariance, $Cov^*(x, y) = E^*(xy) - E^*(x)E^*(y)$. Rearranging, $E^*(xy) = Cov^*(x, y) + E^*(x)E^*(y)$. Since $I(r) > 0$, $E^*(x) > 0$. By No Cream Skimming, $E^*(y) > 0$ and $Cov^*(x, y) \geq 0$. This proves that $E^*(xy) > 0$, so $I'(r) < 0$ as claimed.

Finally, by Lemma B.3, $\underline{\nu}_{\eta\ell}^B = \min \left\{ \bar{\nu}_{\eta\ell}, \bar{\nu}_{\eta\ell} r_{\eta\ell}^{B*}/r_{\eta\ell}^B \right\}$. Substituting for $r_{\eta\ell}^B$, we have the following $\underline{\nu}_{\eta\ell}^B = \min \left\{ \bar{\nu}_{\eta\ell}, \bar{\nu}_{\eta\ell} r_{\eta\ell}^{B*}/\min \left\{ \rho, r_{\eta\ell}^{B*} \right\} \right\} = \bar{\nu}_{\eta\ell}$: bank b does not lend to any borrowers in this group. Moreover, by Lemma B.3, bank b strictly prefers (not) to lend to borrowers whose types ν exceed (respectively, are less than) $\bar{\nu}_{\eta\ell} r_{\eta\ell}^{B*}/r_{\eta\ell}^B$. If $r_{\eta\ell}^{B*} \leq \rho$, then $r_{\eta\ell}^B = \min \left\{ \rho, r_{\eta\ell}^{B*} \right\} = r_{\eta\ell}^{B*}$: bank b is just willing not to bid for the best borrower in this group: the borrower whose private type ν is $\bar{\nu}_{\eta\ell}$. If $r_{\eta\ell}^{B*} > \rho$, then $r_{\eta\ell}^B = \rho < r_{\eta\ell}^{B*}$: bank b strictly prefers not to bid for any borrowers in the group. Q.E.D.

Proof of Lemma B.5: If the bank competes for these borrowers, then (a) by Lemma B.2 it securitizes every borrower who accepts (i.e., $z_{\eta\ell}^B = r_{\eta\ell}^B$) and (b) by Lemma B.3, it outbids bank b for all borrowers in this group: $\underline{\nu}_{\eta\ell}^B = \bar{\nu}_{\eta\ell}$. By differentiating the Lagrangean \mathcal{L} with respect

to $x_{\eta\ell}^B$, one can easily verify that competing for these borrowers (setting $x_{\eta\ell}^B = 1$) raises bank a 's profits if and only if (B.3) holds. Q.E.D.

APPENDIX C. ADDITIONAL MATERIAL FOR CHAPTER 3

Proofs

Proof of Lemma 3.3: Consider the short-term contract $\{B_i\}_{i=1,2}$. The optimal contract requires the bank to punish the defaulting entrepreneur as severe as possible. Thus, due to the limited liability, the payment is $B_1^* = \theta_1$ (and the entrepreneur's payoff $c_1^* = 0$) when the project fails.

If the contract implements a good project ($m = 2$). Hence, the IC constraint (3.3) implies that the entrepreneur's expected utility from the good project $(1 - \pi_2^2) c_1^* + \pi_2^2 c_2^* - v$ is larger than that from the poor one $(1 - \pi_2^1) c_1^* + \pi_2^1 c_2^*$. By the definition $c_2^* = \theta_2 - B_2^*$, I have

$$\theta_2 - B_2^* \geq (\Delta\pi)^{-1} v. \quad (\text{C.1})$$

The IR constraint (3.2) implies that

$$x = \pi_2^2 (\theta_2 - B_2^*) - v. \quad (\text{C.2})$$

Hence, from the IC (C.1) and IR (C.2) constraints, I have $x \geq (\Delta\pi)^{-1} \pi_2^1 v$. But if the contract implements a poor project ($m = 1$). Similarly, the IC constraint (3.3) implies that

$$\theta_2 - B_2^* \leq (\Delta\pi)^{-1} v. \quad (\text{C.3})$$

The IR constraint (3.2) implies that

$$x = \pi_2^1 (\theta_2 - B_2^*). \quad (\text{C.4})$$

Then, from the IC (C.3) and IR (C.4) constraints, I have $x \leq (\Delta\pi)^{-1} \pi_2^1 v$.

The threshold utility \underline{w} to implement the good project is $\underline{w} = (\Delta\pi)^{-1} \pi_2^1 v$. Furthermore, if $x \geq \underline{w}$, from the IR constraint (C.2), the entrepreneur's optimal payoff is $B_2^*(x) = \theta_2 -$

$(\pi_2^2)^{-1}(x+v)$. Since $B_2^*(x) \geq 0$, I have $x \leq \pi_2^2 \theta_2 - v$. Finally, if $x < \underline{w}$, from the IR constraint (C.4), the optimal payoff is $B_2^*(x) = \theta_2 - (\pi_2^1)^{-1}x$. These are the payment arrangement in the lemma. Q.E.D.

Proof of Proposition 3.5: I need the following lemma for the main result.

Lemma C.1 *For $i \in \{1, 2\}$, if $p_i^* > 0$, then it must hold that $x_{i1}^* = \underline{w}$; if $x_{i0}^* > \underline{w}$, then it must hold that $p_i^* = 0$.*

Proof of Lemma C.1: Notice, since the lowest promised utility to implement the good project is \underline{w} by Lemma 3.3, the optimization problem (3.5) implies that

$$\begin{aligned} & \max_{\{p_i, B_i, x_{ij}\}} \sum_{i=1,2} \pi_i^m \sum_{j=0,1} p_{ij} [(B_i - r) + V(x_{ij})] \\ \iff & \max_{\{p_i, B_i, x_{ij}\}} \sum_{i=1,2} \pi_i^m \{B_i + (1 - p_i) [V(x_{i1}) - V(\underline{w})] + V(\underline{w})\} - r. \end{aligned}$$

But this is equivalent to the following problem,

$$\max_{\{p_i, x_{i1}\}} (1 - p_i) [V(x_{i1}) - V(\underline{w})] \quad (\text{C.5})$$

subject to $p_i \in [0, 1]$ and $x_{i1} \geq \underline{w}$ for $i \in \{1, 2\}$. Therefore, if $p_i^* > 0$, it must be the case that $V(x_{i1}^*) = V(\underline{w})$ since $V'(x) < 0$, or $x_{i1}^* = \underline{w}$. If $x_{i1}^* > \underline{w}$, then I have $V(x_{i1}) - V(\underline{w}) > 0$, which implies $p_{i1}^* = 1$ and thus $p_i^* = 0$. Q.E.D.

It is straightforward to establish two preliminary results. When the credit line is terminated, the entrepreneur cannot refinance the project due to his credit history. It is optimal to set the promised utility to zero if the credit line is cut off. So the expected utility to the entrepreneur is $x_{i0}^* = 0$ for $i = 1, 2$ when the bank cuts off the credit line. And the optimal contract requires the bank to punish the defaulters as severe as possible, and $B_1^* = \theta_1$ and $c_1^* = 0$ due to limited liability.

For an optimal long-term loan contract which implements the good project, the IR (3.6) and IC (3.7) constraints must hold,

$$(\text{IR}) \quad \sum_{i=1,2} \pi_i^2 \sum_{j=0,1} p_{ij} (c_i + x_{ij}) - v = X, \quad (\text{C.6})$$

$$(IC) \sum_{i=1,2} \pi_i^2 \sum_{j=0,1} p_{ij} (c_i + x_{ij}) - v \geq \sum_{i=1,2} \pi_i^1 \sum_{j=0,1} p_{ij} (c_i + x_{ij}). \quad (C.7)$$

Recall that $p_{i0} = p_i$, $p_{i1} = 1 - p_i$, and $c_i = \theta_i - B_i$. By Lemma C.1, I have $x_{21}^* = \underline{w}$ when $p_2^* > 0$. Then, in the second period, the entrepreneur is indifferent with the two projects. However, for any $\varepsilon > 0$, if the bank sets $x_{21}^* = \underline{w} + \varepsilon$, the entrepreneur will prefer the good project and the bank can make more profits. It is a contradiction. Therefore, $p_2^* = 0$ solve the optimization problem. Now, if $p_1^* > 0$, I have $x_{11}^* = \underline{w}$ by Lemma C.1. Then, given the expected utility X , the IR (C.6) constraint could be written as

$$\pi_1^2 (1 - p_1^*) \underline{w} + \pi_2^2 (\theta_2 - B_2^* + x_{21}^*) = X + v. \quad (C.8)$$

And the IC (C.7) constraint must be binding

$$(\theta_2 - B_2^* + x_{21}^*) - (1 - p_1^*) \underline{w} = (\Delta\pi)^{-1} v. \quad (C.9)$$

From the two equations (C.8) and (C.9), I get

$$\theta_2 - B_2^* + x_{21}^* = (X - \underline{w}) + (\Delta\pi)^{-1} v, \quad (C.10)$$

$$p_1^* = 1 - \underline{w}^{-1} \left[X + v - (\Delta\pi)^{-1} \pi_2^2 v \right] = 2 - \underline{w}^{-1} X. \quad (C.11)$$

Since $p_1^* \in [0, 1]$, the bank's optimal termination policy comes directly from the equation (C.11).

Finally, from the equation (C.10), I have the loan plan,

$$x_{21}^* - B_2^* = X + (\Delta\pi)^{-1} (1 - \pi_2^1) v - \theta_2.$$

Q.E.D.

Proof of Proposition 3.7: For part 1, I only consider old entrepreneurs. I compare the new project and the poor project. Assume the interest rate for a loan is R , so the expected return is $\pi_2^1(\omega_2)(\theta_2(\omega_2) - R)$ for the new project. Since $\pi_2^1(\omega_2) < \pi_2^1(\omega_1)$, the bank's expected return decreases if the entrepreneur chooses the new project

$$(1 - \pi_2^1(\omega_2))\theta_1 + \pi_2^1(\omega_2)R - r < (1 - \pi_2^1(\omega_1))\theta_1 + \pi_2^1(\omega_1)R - r. \quad (C.12)$$

From (C.12) and the new project and the poor project have the same expected return, the entrepreneur's expected return increases

$$\pi_2^1(\omega_2)(\theta_2(\omega_2) - R) > \pi_2^1(\omega_1)(\theta_2(\omega_1) - R). \quad (C.13)$$

Moreover, given the optimal contracts, the entrepreneur is indifferent between the poor project and the good project in state ω_1 . And thus he will prefer the new project over the good project.

For part 2a, by Proposition 3.7, banks need to give more incentive rents to entrepreneurs to choose the good project in state ω_2 . And from Lemma 3.3 and $\pi_2^1(\omega_2) < \pi_2^1(\omega_1)$, I have $\underline{w}(\omega_2) > \underline{w}(\omega_1)$, which implies

$$\mu(r^*(\omega_2) | \underline{w}(\omega_2)) > \mu(r^*(\omega_1) | \underline{w}(\omega_1)).$$

Finally, for part 2b, assume there is only an infinitesimal change in the two states ω_1 and ω_2 so that the threshold $\underline{w}(\omega_t)$ changes continuously. Omit the time subscript without confusing. Total differentiating equilibrium conditions (3.13) and (3.14) gives

$$u''(h-s) ds + dr = 0,$$

$$(1-\eta)(\underline{w})^2 ds + \eta\alpha\pi_1^2 \underline{w} dr = -\eta\alpha\pi_1^2 (\bar{\theta}^2 - v - r) d\underline{w}.$$

That is

$$\begin{pmatrix} u''(h-s) & 1 \\ (1-\eta)(\underline{w})^2 & \eta\alpha\pi_1^2 \underline{w} \end{pmatrix} \begin{pmatrix} \frac{ds}{d\underline{w}} \\ \frac{dr}{d\underline{w}} \end{pmatrix} = \begin{pmatrix} 0 \\ -\eta\alpha\pi_1^2 (\bar{\theta}^2 - v - r) \end{pmatrix}. \quad (\text{C.14})$$

And by the Cramer's rule to (C.14), I have

$$\frac{ds}{d\underline{w}} = [\det(A)]^{-1} \eta\alpha\pi_1^2 (\bar{\theta}^2 - v - r) < 0,$$

$$\frac{dr}{d\underline{w}} = -[\det(A)]^{-1} u''(h-s) \eta\alpha\pi_1^2 (\bar{\theta}^2 - v - r) < 0,$$

where

$$A = \begin{pmatrix} u''(h-s) & 1 \\ (1-\eta)(\underline{w})^2 & \eta\alpha\pi_1^2 \underline{w} \end{pmatrix},$$

with $\det(A) = u''(h-s) \eta\alpha\pi_1^2 \underline{w} - (1-\eta)(\underline{w})^2 < 0$ and $u''(\cdot) < 0$. Q.E.D.

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